ESTIMATING DEFAULT PROBABILITIES FROM SOUTH AFRICAN BOND MARK TO MARKET DATA

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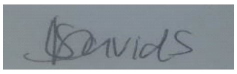
16227638	Recoverable Signature X  Signed by: Sheree Davids
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Abstract

The increase in awareness of counterparty credit risk and international financial reporting standards has led to the requirement of estimates for the default probability of counterparties in over the counter derivative transactions. Default probabilities are necessary for the calculation credit value adjustments (CVA), which can be seen as the quantification of counterparty credit risk. In this research paper different methods and models for estimating default probabilities is reviewed and a detailed summary of the application of these models in a South African context is given. It was found that credit risk models can, to an extent, be used as a reflection of credit risk. The methodology followed using South African bond mark to market data to obtain a matrix of credit spreads for different rating classes is then outlined and corresponding default probabilities for the different rating classes are then calculated. The methodology however is not completely general and the obtained credit spreads are higher than that of previous studies. Overall it was found that South African bond mark to market data can be used for inferring default probabilities for the purpose of CVA calculations.

Key words:

Credit valuation adjustments; Default probabilities; Credit spreads; Credit ratings; South African bond market

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List of abbreviations and/or acronyms

CCR	Counterparty credit risk
OTC	Over the counter
CVA	Credit valuation adjustment
CSA	Credit support annex
IFRS	International Financial Reporting Standards
CDS	Credit default swap
EDF	Expected default frequency
S&P	Standard & Poor
BESA	Bond Exchange of South Africa
STvD	Shimko, Tejima and van Deventer
CAPM	Capital Asset Pricing Model
JSE	Johannesburg Stock Exchange

CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

As opposed to market risk, credit risk is still a developing field and there are fewer common practices. Credit risk is the risk arising from the uncertainty with regard to fulfilment of a debtor or counterparty's contractual obligations in terms of interest and principle payments (Gregory, 2012a: 10). Counterparty credit risk (CCR), also known as default risk, is the credit risk that exists between over the counter (OTC) derivative counterparties (Gregory, 2012a: 18). The measurement of default risk and understanding probabilities of default is therefore important to the general market. Every derivative position held, where the contract is an asset, is exposed to default risk unless it is fully mitigated (Hull, 2009: 500). Most OTC positions that do not have a credit support annex (CSA) in place are exposed to CCR and fair value adjustments need to be made (Gregory, 2012a: 62-63) to incorporate CCR. One of these adjustments, credit valuation adjustment (CVA), which can be seen as the measurement of CCR, is now more often than not considered in the pricing of derivative trades as well as financial reporting. For these adjustments to be made certain values need to be estimated, one of which is the default probability of the counterparty.

1.2 BACKGROUND/RATIONALE

Following the September 2008 financial crisis, which was mainly a result of insufficient financial regulation, it was clear that no counterparty could be regarded as completely risk free or too big to fail. Counterparty credit risk became increasingly prevalent in global financial markets (Gregory, 2012a: 6). International Financial Reporting Standards (IFRS) 13, effective from 1 January 2013 states

> "The entity shall include the effect of the entity's net exposure to the credit risk of that or the counterparty's net exposure to the credit risk of the entity in the fair value measurement when market participants would take into account any existing arrangements that mitigate credit risk exposure in the event of default" (Gregory, 2012b: 4).

With the increase in general awareness of CCR and changes in financial reporting regulation, the market has moved away from risk-free pricing and started incorporating CCR within their derivative valuation (Ernst & Young, 2014).

Counterparty risk and CVA is a relatively new topic and many financial institutions are still in the process of integrating it into their existing risk management and reporting structures. Default risk modelling is an integral part of CVA calculations as counterparties' default probabilities are needed for CVA calculations. A survey conducted by Deloitte (2013) found a consensus, that because of the accounting standards, CVA quantification should rather use market implied parameters than historical parameters. In order to implement these regulations, various models need to be constructed. One of these would be a model that estimates risk neutral default probabilities, which is a parameter obtained from observed market data. The necessary market data also needs to be modelled to obtain values for unobservable points.

1.3 PROBLEM STATEMENT

The changes in financial reporting regulations imply that non-banks also have to incorporate fair value adjustments like CVA into their derivative valuation. Therefore a simple and efficient model is necessary to estimate default probabilities that correspond to the defaultable entity. It is therefore the aim of this research to model the term structure of risk neutral default probabilities for the different credit ratings that can be used for CVA calculations.

Defining credit spreads from the premiums of single-name credit default swaps (CDSs) instead of bond yields compared to some benchmark would give a more accurate measure of CCR, but CDS data is complex and not readily available. Using bond market data should give a similar spread to the spreads obtained from the CDS market Gregory (2012a: 215). Therefore for non-banks the bond mark to market data would be easier to obtain and interpret to use for defining credit spreads. Obtaining a model that accurately estimates risk neutral default probabilities from bond mark to market data would imply that it could be easily implemented for CVA modelling and used for valuation of derivatives.

Theoretically, the plot of the term structure of default probability should be an upward sloping curve, because the marginal probability of default increases over time (Gregory, 2012a: 206). The fitted curves for the models of the different credit ratings should more or less have the same shape but with the average probability of default of corporate bonds with higher credit ratings being lower, because bonds with lower credit ratings run a greater risk of defaulting.

1.4 RESEARCH DESIGN

This research paper has both qualitative and quantitative aspects. The qualitative part, which is the majority of this paper, is the review of different methods for inferring or estimating default probabilities with specific reference to methods using market data and more specifically credit spreads obtained from market data.

In Chapter 2 the relevant literature regarding default probabilities is discussed in detail. First there is referred to CVA and which measure of default probability is necessary for its quantification. Thereafter different classes of models, structural and reduced form, for estimation of risk-neutral default probabilities and credit spreads are reviewed with regard to the underlying theory and practical implementation of the models or methods.

The relationship between corporate bond spreads and credit default swap spreads is also discussed and why corporate bond spreads can be used for estimation of risk-neutral default probabilities and ultimately the quantification of CVA. Reference is also made to different methods for modelling the term structure of credit spreads on corporate bonds form observed market data through interpolation and extrapolation. Applications of the Merton model on South African data as well as results of the empirical studies are given and interpreted.

Thereafter follows Chapter 3, which describes the methodology followed for obtaining credit spreads for the different rating classes and all unobservable maturities from South African bond mark to market data. The credit spreads for the different rating classes are then used for estimation of the default probability for the different rating classes.

In Chapter 4 the results obtained from following the methodology in Chapter 3. The methods were applied to consecutive months of data to test whether it is not just suitable for the particular data set it was developed from. It is then also compared with previous results from studies on the South African market to test whether the model delivers results which are in line with expectations and delivers predictions that can be used for the purpose of CVA quantification.

A summary of the results and findings as well as overall conclusions can then be found in Chapter 5. It also includes the limitations of this research and recommendations for further study.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter the literature on the factors necessary for quantification of credit valuation adjustments (CVA), which includes the construction of credit spreads and estimation of default probabilities will be discussed and interpreted. There is also referred to how corporate bond spreads can be linked to credit default swap spreads and the use thereof for estimating default probabilities. The application of credit risk models and default probabilities in a South African context are also discussed.

2.2 CREDIT VALUATION ADJUSTMENTS

Counterparty credit risk (CCR) is a representation of market risk; in the form of the expected exposure of the financial instrument or derivative, and credit risk which defines the credit quality of the counterparty. A counterparty with large exposure and small default probability might be less desirable than a counterparty with small exposure and large default probability. CVA is precise quantification of CCR which makes it comparable and able to distinguish between the two before mentioned scenarios (Gregory, 2012a: 18).

Global regulation after the 2008 financial crisis seems to view CVA as being a necessary mark to market trading book item; which is regularly updated and calculated from market variables, rather than a banking book component calculated form historical data, which is not updated and held to maturity, as it was seen in the past (Gregory, 2012a: 19). This mark to market value of CVA should be presented along with the over the counter (OTC) derivative it was derived from. When valuing an OTC derivative CCR must be included, but it is possible for the value to be divided into separate components, which implies that a transaction and CCR accompanied with it can be priced and risk-managed separately (Gregory, 2012a: 242).

$$Risky\ value = risk\ free\ value - CVA \qquad (2.1)$$

The above relation is not linear because CVA is not additive for individual transactions due to the mitigation of credit risk through netting and collateralization. Netting is a clause in OTC derivative contracts which states that if a counterparty defaults on one contract with a company, it defaults on all outstanding contracts with the specific company. Collateralization is an agreement which typically states that: the contracts between a financial institution and a company should be valued periodically and if the total value is higher than the specified threshold, the collateral posted should be equal to the difference between the value and the threshold (Hull, 2009: 502-503).

Under certain simplifying assumptions; the institution cannot default, risk-free valuation is possible and the probability of default and exposure are independent, Gregory (2012a) derives the following equation for CVA:

$$CVA \approx (1 - Rec) \sum_{i=1}^{m} DF(t_i) EE(t_i) PD(t_{i-1}, t_i) \qquad (2.2)$$

Where $(1 - Rec)$ is the loss given default, DF is the relevant risk-free discount factor, EE is the expected exposure for the relevant dates and PD is the marginal default probability between the relevant dates (Gregory, 2012a: 243).

IFRS 13, which is a set of international accounting standards regarding the reporting of financial instruments and transactions, requires that CCR be incorporated into derivative valuations, which are credit and debit valuation adjustments for derivatives. The accounting literature does not prescribe which methods should be used for the quantification of these adjustments and therefore various methods are applied in practice, which are influenced by factors such as cost and availability of technology and data, number of contracts and credit risk mitigation (Ernst & Young, 2014). It is also required that the use of observable data should be maximised, as fair value is a market based measurement. This implies that the use of current credit spreads observed in the market as a source of credit risk data, rather than historical data, is ranked higher.

Credit default swap (CDS) spreads provide a good indication of the counterparty's creditworthiness from the market participants' point of view, but CDS spreads are not always available to smaller companies and other observable indicators of creditworthiness like publicly traded debt should be used. In case of absence of observable indicators, a combination of factors can be used for indication of creditworthiness for CVA quantification. All selection basis of the input and all assumptions made should be documented as part of the analysis according to IFRS 13 (Ernst & Young, 2014).

2.3 DEFAULT PROBABILITIES

The research on estimating default probabilities originated with the development of theory for the valuation of bonds with a significant probability of default. It can be categorised into two different classes of models based on different frameworks; structural models and reduced form models (Arora *et al,* 2005).

For investment grade bonds it is typical that default probabilities are an increasing function of time. The reasoning behind this is that initially the issuer is regarded as creditworthy, but as time increases the creditworthiness declines because of the possibility of a downfall in financial health. For lower credit rating the converse is true, because the issuer's financial health is initially questionable but as time passes and the issuer survives it shows prospects of an improvement of financial health (Hull, 2009: 420).

2.3.1 Real World vs Risk Neutral

There is differentiation between real-world or historical default probabilities and risk neutral default probabilities. Real world default probabilities are obtained from historical data analysis of actual defaults and published by rating agencies, whereas risk neutral default probabilities are inferred from market data. Risk neutral default probabilities are so called because of the assumption that expected default losses can be discounted at the risk free rate under the risk neutral valuation principle (Hull, 2009: 496-497). There is no contradiction between what the two values represent but they have different applications.

Many empirical studies suggest that the difference between spreads on corporate bonds and the part of the spread that can be seen as compensation for default is significant, where with risk-neutral default probabilities the entire spreads is seen as compensation for default. Hull, Predescu and White (2004) referred to Altman (1989) being one the first to note the difference between historical default data and corporate bond spreads. Elton (2001), Haung and Haung (2002) and Amato (2003) are some of the studies done on the topic which conclude that together with the actual/historical default probability there is a default risk premium, liquidity premium as well as a tax premium that accounts for the credit spread. Hull *et al* (2004) also referred to systematic risk contributing to the excess return, noting that the default of bonds do not occur independently. This can be seen when comparing default probabilities in different time periods, showing that there are periods of time when it is low and periods where it is high.

Gregory (2012a: 198) describes risk-neutral default probabilities as a reflection of the market price of default risk rather than estimates of actual default probabilities. Risk neutral probabilities, observed from credit spreads in the market are used for the valuation of credit derivatives where real-world default probabilities are relevant for assessments of risk like bank capital requirements (Hull *et al*, 2004: 1). Therefore risk neutral default probabilities would be used for calculations of CVA.

2.3.2 Structural Models

The earliest approaches to modelling default and credit risk were based on Black-Scholes option pricing methodology. Merton (1974) recognised that the Black-Scholes model's approach could be used for corporate liability valuation, therefore linking default probability to the counterparty's asset value. An equation used for valuation was developed and applied to corporate debt to obtain a formula for the risk structure of interest rates, where 'risk' is defined as gains or losses due to changes in default probability, which is in essence the term structure of credit spreads.

The risk premium $R(\tau) - r$, defined as the risk structure of interest rates developed by Merton (1974) is:

$$R(\tau) - r = \frac{-1}{\tau} log\left\{\Phi[h_2(d, \sigma^2\tau)] + \frac{1}{d}\Phi[h_1(d, \sigma^2\tau)]\right\} \qquad (2.3)$$

Where $R(\tau)$ is the yield to maturity of the zero-coupon debt issue with value F and face value B; $e^{-R(\tau)\tau} = F[V, \tau]/B$ and r is the risk-free rate.

The above equation was developed from the value of the bond F. Assuming the firm's value V follows the stochastic process $dV = (\alpha V - C)dt + \sigma V dz$, B is the amount owed to the bondholders at time T and τ is the time to maturity. The formula for F:

$$F[V, \tau] = Be^{-r\tau}\left\{\Phi[h_2(d, \sigma^2\tau)] + \frac{1}{d}\Phi[h_1(d, \sigma^2\tau)]\right\}$$

Where $d = Be^{-r\tau}/V$,

$h_1(d, \sigma^2\tau) = -\left[\frac{1}{2}\sigma^2\tau - \log(d)\right]/\sigma\sqrt{\tau}$ and

$h_2(d, \sigma^2\tau) = -\left[\frac{1}{2}\sigma^2\tau + \log(d)\right]/\sigma\sqrt{\tau}$.

Equation (2.3) is dependent on the firm's volatility σ and $d = Be^{-r\tau}/V$, which can be seen as the debt-to-firm value or leverage ratio, where the debt is valued at r, the riskless rate. It is therefore a biased estimate of the actual leverage ratio, which would be lower.

Default probabilities can then be inferred from the Merton (1974) model. Default will occur if at time T the firm's value is less than what is owed to the bondholders, thus $V(T) < B$ and the probability of default can be obtained as follows from the underlying Black Scholes assumptions that the Merton Model is based on (Frey, 2010:16). Where μ and σ refer to the mean and volatility rates of the stochastic process of the firms value V; $dV = \mu V dt + \sigma V dz$.

$$P(V(T) < B) = \Phi\left[\frac{\ln\left(\frac{B}{V(0)}\right) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right] \qquad (2.4)$$

The framework of Merton (1974) set the basis for many models and studies. One of which, Delianedis and Geske (1998), highlighted the importance of default probabilities to the credit markets, showing that risk neutral default probabilities serve as upper bounds to estimates of historical default probabilities and contain credit rating transition information. This class of models model the asset and liability structure of the counterparty and infer default probabilities from those models. These structural form models addressed qualitative aspects of pricing credit risk, but had difficulty with its practical implementation.

Moody's KMV extended the Merton approach and relaxed some of the assumptions for better empirical performance (Frey, 2010: 21) with the aim of measuring default by producing 1 year default probabilities defined as expected default frequency (EDF). The model is also extended to value corporate securities and produces robust predictions of the credit spreads on said securities (Arora *et al*, 2005).

$$EDF_{Merton} = 1 - \Phi\left[\frac{\ln V(0) - \ln B + (\mu - \frac{1}{2}\sigma^2)}{\sigma}\right] \quad (2.5)$$

Where V is determined from the firm's equity value using Merton's model and the volatility of the assets is denoted by σ, determined from the firm's equity data.

2.3.3 Reduced Form Models

The above mentioned structural models were difficult to implement practically and an alternative, simpler approach was sought after and developed. This was known as the reduced form approach, where bankruptcy was not explicitly dependent on the underlying assets value of the counterparty. With this class of models, default was modelled under the assumption that at each instant there is some probability that a bond could default; default is often seen as a hazard-rate process. These models were more mathematically manageable than structural models, making it easier to implement practically (Duffee, 1999: 198).

An example of such a reduced form model is the model developed by Jarrow *et al* (1997). It incorporated credit quality as an indicator of default likelihood of the counterparty, more explicitly using the firm's credit rating as a measure of the credit quality of the firm for modelling the term structure of credit risk spreads. In this model, bankruptcy is characterised as a Markov process independent of the default free term structure. The time-homogenous Markov chain, with state space representing K different credit classes specified by transition matrix Q, represents the default time.

$$Q = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1K} \\ q_{21} & q_{22} & \cdots & q_{2K} \\ \vdots & \vdots & & \vdots \\ q_{K-1,1} & q_{K-1,2} & \cdots & q_{K-1,K} \\ 0 & 0 & \cdots & 1 \end{pmatrix} \quad (2.6)$$

The transition probability from time t to t+1 is denoted by $q_{ij}(t, t+1)$ can be estimated by the equivalent martingale probabilities when no arbitrage and complete markets are assumed and the transition probability matrix follows

$$\tilde{Q}_{t,t+1} = \begin{pmatrix} \tilde{q}_{11}(t,t+1) & \tilde{q}_{12}(t,t+1) & \cdots & \tilde{q}_{1K}(t,t+1) \\ \tilde{q}_{21}(t,t+1) & \tilde{q}_{22}(t,t+1) & \cdots & \tilde{q}_{2K}(t,t+1) \\ \vdots & \vdots & & \vdots \\ \tilde{q}_{K-1,1}(t,t+1) & \tilde{q}_{K-1,2}(t,t+1) & \cdots & \tilde{Q}_{K-1,K}(t,t+1) \\ 0 & 0 & \cdots & 1 \end{pmatrix} \quad (2.7)$$

The martingale probabilities then depend on the history of the process up until time t, thus the process does not have to be Markov. The probabilities can be written as $\tilde{q}_{ij}(t,t+1) = \pi_i(t)q_{ij}$, where $\pi_i(t)$ is a function of time, such that satisfies the transition probability conditions. The interpretation of $\pi_i(t)$ is, it is the risk premium on debt issues of the specific credit class. Now the n-step transition probability matrix is obtained by taking the product; $\tilde{Q}_{0,n} = \tilde{Q}_{0,1}\tilde{Q}_{1,2} \dots \tilde{Q}_{n-1,n}$. The probability of default after time T, that is $\tilde{Q}_t(\tau^* > T)$ can now be calculated from this structure. If the firm is in state i at time t and τ^* is the time that the process reaches state K, which represents bankruptcy, the probability is:

$$\tilde{Q}_t(\tau^* > T) = \sum_{j \neq K} \tilde{q}_{ij}(t,T) = 1 - \tilde{q}_{iK}(t,T) \quad (2.8)$$

Duffie and Singleton (1999) introduced a new approach to modelling defaultable bonds' term structures; losses at default were parameterised in terms of the reduction in market value at the occurrence of default. Duffee (1999) modelled default probability as a translated square root diffusion process, allowing for correlation with default free interest rates and followed the framework of Duffie and Singleton (1999) to test whether reduced models could price credit risk accurately. It was found that the model could explain the corporate bond yield data well but could not explain all fluctuations and as the credit quality changed parameter instability was found.

Hull and White (2000) developed a reduced form model for the valuation of credit default swaps which included estimation of default probabilities of a specific reference entity. They defined the difference between the value of a corporate bond and a similar Treasury bond as the present value of the cost of default. This relationship was then used to infer default probabilities by calculating the present value of a number of different bonds issued by the entity and making an assumption about recovery. The inference can be described by the following equation:

$$PV(cost\ of\ default) = V(t).PD.e^{-rt} \quad (2.9)$$

Where $V(t)$ denotes the loss incurred at time t, r is the corresponding yield of the Treasury bond and PD denotes the probability of default.

Hull *et al* (2004) examined credit spreads on corporate bonds and the difference between real world default intensities and the risk neutral default intensities implied from bond prices. Default was governed by Poisson process with constant default intensity or hazard rate (2.10). The default intensity used in the study was the excess corporate bond yield as a fraction of loss given default $(1 - R)$ as an approximation (2.11). The underlying idea under the default intensity estimate is that excess return (spread) is compensation for the cost of default. Gregory (2012a) provides theoretical justification for this default intensity approximation which can be found in Appendix B.

$$F(u) = 1 - \exp[-hu] \quad (2.10)$$

Where $h \approx \frac{Spread}{1-R}$ (2.11)

Substituting the approximation of the hazard rate (2.10) into (2.9) then yields the following equation:

$$F(u) = 1 - \exp\left[-\frac{Spread}{1-R}u\right] \quad (2.12)$$

Equation (2.11) represents the cumulative default probability over time period u. The marginal default probability applicable for the period of time from t_{i-1} to t_i, which would be used for calculations of CVA then follows,

$$q(t_{i-1}, t_i) \approx \exp\left[-\frac{Spread_{t_{i-1}}}{1-R}t_{i-1}\right] - \exp\left[-\frac{Spread_{t_i}}{1-R}t_i\right] \quad (2.13)$$

The above approximation is found by taking the difference between the cumulative default probabilities at times t_{i-1} and t_i and is specified as such under Basel III for the calculation of CVA. The approximation however, does not take the credit spread curve shape into account and the more curved the shape the worse the approximation becomes (Gregory, 2012a: 206).

2.4 CREDIT SPREADS

2.4.1 Credit Default Swap Spreads

The benchmark risk free rate used by participants in credit markets can be estimated from the CDS market (Hull, 2009: 494), but the CDS market data is complex and not readily available. It is possible to show that a position in a fixed rate bond and interest rate swap is equivalent to a CDS position as shown by Duffie (1998). An asset swap is a derivative which can be seen as a portfolio consisting of a fixed rate bond and a fixed paying interest rate swap. Markets for fixed rate bonds have sometimes, not been as liquid as the markets for corresponding asset swaps and therefore asset swap spreads are often used for pricing default swaps. Duffie (1998) shows how asset swap spreads together with default free rates can be used to estimate default swap spreads.

Suppose the asset swap spread on the default-free floating rate is quoted at $\hat{S}$, the fixed rate on the underlying bond is C and the default-free swap rate is C^*. Therefore the fixed swap rate on the underlying interest rate swap is $C - \hat{S}$. The fixed rate spread F over the default-free coupon rate of a bond that has the same credit quality as the underlying bond can now be determined. It is calculated from the price of a portfolio consisting of the asset swap and a short position in a portfolio including a fixed rate bond with the same credit quality as the underlying bond of the asset swap and an at-market interest rate swap. The worth of this portfolio is, when the price of the defaultable annuity is A and the default-free annuity with the same maturity: A^*,

$$1 - 1 = 0 = A(C - F + C^*) + A^*(C^* - C + \hat{S}\,) \tag{2.14}$$

From which the value of the fixed rate spread F follows:

$$F = C - C^* - \frac{A^*}{A}(C - \hat{S} - C^*) \tag{2.15}$$

This par fixed rate spread F is now approximately the same as the par floating rate spread S, which is considered the basis for the default-swap spread. Therefore the par asset-swap spread is approximately equal to the default-swap spread S.

The above mentioned relationship implies that using bond market data should give a similar spread to the spreads obtained from the CDS market, although the relationship is imperfect due to certain factors; the wrong-way risk of CDSs, fixed rate bonds often trade above and below par and CDSs are indexed to the par value of a bond, accrued interest is not paid in the event of default, but CDSs make provision for accrued interest and other further technicalities (Gregory, 2012a: 215).

Spreads on corporate bonds, which is the difference between yields on corporate bonds and government bonds or treasuries which are presumed to be free of default risk, are generally interpreted as compensation for credit risk. Therefore corporate bond spreads contain information on the possibility and likelihood of default of the counterparty. However, as mentioned before, many empirical studies have shown that actual default risk only accounts for a portion of the credit spread.

2.4.2 Term Structure

There is extensive empirical research done on the term structure of credit spreads on corporate bonds and many are studies done on constructing credit spreads from historical default probabilities published by rating agencies. A specific area of interest is the shape of the credit spread term structure. Altman (1989) also showed that as the credit rating declined, the higher the excess return on the corporate bonds was.

The Markov model of Jarrow *et al* (1997) estimated credit spreads and found upward sloping credit spreads for investment grade bonds for short to medium term maturities. Although there were significant differences between the slope of the credit spreads for the different rating classes, with the slope increasing as the credit quality decreases.

Fons (1994) also found the general consensus that issuers with higher credit quality have narrower credit spreads that widen with maturity while lower rated issuers have wider credit spreads that become narrower with longer maturities.

Altman (1989), Fons (1994) and Jarrow *et al* (1997) all found that for investment grade securities, over short to medium term maturities, the credit spread is upward sloping. For speculative grade securities however, the credit spread tends to be downward sloping or humped but there are still contradicting results from different models (Bohn, 1999: 1). Bohn (1999) conducted a study regarding the term structure of credit spreads for low credit quality bonds in response to the conflicting results and found downward sloping and humped shaped term structures for low credit quality bonds.

The slope and shape of the credit spread tends to differ as credit quality decreases. In the Merton (1974) framework the determinant of the slope is the estimated leverage ratio which would lead to different slopes for different credit classes.

Joint estimation approach was introduced by Houweling *et al* (2001) because simple estimation procedures could lead to twisting credit spread curves for different markets or peer comparisons which are not realistic and causes complications when used in financial models. They find smoother curves when using this joint estimation method as opposed to traditional methods. Jankowitsch and Pichler (2004) also considered a different approach of credit spread estimation, instead of the subtracting a risk-free term structure from the risky term structure, when different markets are compared.

Benzschawel and Assing (2012) used historical default probabilities and added a risk premium to construct credit spreads and found upward sloping term structures for short maturity bonds for all credit classes.

2.4.3 Interpolation and Extrapolation

Various methods are used for the interpolation and extrapolation of observed credit spreads but the use of fitting parametric curves and polynomial splines seems to be common practice.

Nelson and Siegel (1987) approach to fitting a parametric curve and using regression to estimate the necessary parameters is used for yield curve modelling. It is a simple yet parsimonious model (2.3) that is flexible enough to accommodate different curve shapes and is therefore useful for credit spread modelling.

$$r(t) = \beta_0 + \beta_1 \left[\frac{1-\exp\left(-\frac{t}{\tau}\right)}{\frac{t}{\tau}}\right] + \beta_2 \left[\frac{1-\exp\left(-\frac{t}{\tau}\right)}{\frac{t}{\tau}} - \exp\left(-\frac{t}{\tau}\right)\right] \quad (2.16)$$

A cubic spline is a piecewise function fitting a third degree polynomial (2.4) between two points in the data set; x_i and x_{i+1} using the following equation:

$$s_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i \quad (2.17)$$

Cubic- and polynomial splines in general are useful for data sets where the space between consecutive data points is equal and do not deviate from the expected shape of the curve as the obtained curve goes through all the data points.

Hagan and West (2006) investigated different interpolation methods for curve construction used in financial markets. Raw interpolation (2.5) was one of the methods.

$$r(\tau) = \frac{\tau - t_i}{t_{i+1} - t_i} \frac{t_{i+1}}{\tau} r_{i+1} + \frac{t_{i+1} - \tau}{t_{i+1} - t_i} \frac{t_i}{\tau} r_i \qquad (2.18)$$

Orr (1997) fit the following curve, where t denotes the term of the yield, for extrapolating yield predictions to fit entire yield curves and performed regression analysis on the sets of parameters generated for different months to determine the relationship between short and medium term yields.

$$y_t = (a_1 + a_2 t).\mathrm{e}^{a_3 \mathrm{t}} + a_4 \qquad (2.19)$$

2.4.4 Callable bonds and Floating rate bonds

A callable bond is a bond where the issuer has an option to not repay the face value of the bond at a predetermined value before the maturity of the bond. Bonds with embedded call options have different yield term structures than that of non-callable bonds. The spreads of callable corporate bond yields over non-callable Treasury bond yields depends on the callability of the bond (Duffee, 1998: 2225).

Callable bonds also have negative convexity at par, while non-callable bonds always have positive convexity (Finance Train, 2014). Positive convexity implies that price increases as a result of falling bond yields are greater than price decreases which are caused by a rise in bond yields. In the previously mentioned models in section 2.3, for bond valuation and credit spread construction, callable bonds were considered separately. Duffie and Singleton (1999) distinguished between the valuation of callable and non-callable bonds, this is done because assumptions have to be made about the issuer’s call policy. Duffee (1996) as well as Duffee (1999) excluded callable bonds from the empirical study done on corporate yield spreads. The Merton (1974) model was also developed on the basis that the debt or face value of the bond has to be paid, but can be extended for the case of callable bonds.

Floating rate bonds are bonds for which the coupons are not fixed but rather refixed with reference to a specific benchmark (Rajwade, 2005: 12). The valuation of floating rate bonds is therefore more mathematically complex than the valuation of fixed rate bonds, because of the unknown value of the coupons. The floating rate is defined as a spread over some reference rate and it is especially complicated when the desired spread differs from the contracted spread (Rajwade, 2005: 12).

In many empirical studies the only bonds with no variation in coupon payments or maturity and without imbedded options are considered, such as Duffee (1999), Elton *et al* (2001), Bedendo, Cathcart and El-Jahel (2004). These are known as straight bonds or plain vanilla bonds.

2.6 SOUTH AFRICAN CONTEXT

Historically South African companies in general do not have high default probabilities which could be a reflection of South African firms not making use of financial leverage as much as international firms (Holman, Van Breda, and Correia, 2011: 1). Therefore credit risk models developed from international data might deliver different results when applied to South African data.

2.6.1 Credit Ratings

Corporate bonds need to be assessed in term of creditworthiness unlike treasury bonds that have government guarantee and are considered risk-free. Corporate debt ratings are thus an assessment of the ability of the issuer to meet its obligations, in term of interest and principal payments, and indicate the level of default risk corresponding to the corporate debt (Ndlovu, 2002: 50).

Bonds are rated by independent, internationally recognised agencies such as Standard & Poor (S&P), Fitch and Moody's and Bond Exchange of South Africa (BESA) support ratings from these agencies. The highest credit rating is AAA, which indicates high credit quality and financial health, whereas the lowest rating, C, indicates poor credit quality. Further the ratings are also separated into two categories; investment grade, which are ratings BBB or Baa for Moody's and above, and speculative grade, which are ratings BB or Ba and lower (Ndlovu, 2002: 51-52).

Moody's assign ratings based on forward looking opinions of the credit risk associated with issuers and financial obligations (Table 2.1). They append numerical modifiers 1, 2 and 3 to each rating to indicate the rank of the obligation within the rating classification, with 1 indicating the higher end, 2 the mid-range and 3 the lower end. These modifiers are equivalent to the + an - that are appended to S&P and Fitch ratings.

Moody's also have national long term ratings which are an indication of the creditworthiness of the issuer or obligation relative to others within the given country (Table 2.2). The last two letters indicate the country of the issuer i.e. Aaa.za for South Africa.

Table 2.1: Moody’s Global scale ratings descriptions

Global Long-Term Rating Scale	
Aaa	Obligations rated Aaa are judged to be of the highest quality, subject to the lowest level of credit risk.
Aa	Obligations rated Aa are judged to be of high quality and are subject to very low credit risk.
A	Obligations rated A are judged to be upper-medium grade and are subject to low credit risk.
Baa	Obligations rated Baa are judged to be medium-grade and subject to moderate credit risk and as such may possess certain speculative characteristics.
Ba	Obligations rated Ba are judged to be speculative and are subject to substantial credit risk.
B	Obligations rated B are considered speculative and are subject to high credit risk.
Caa	Obligations rated Caa are judged to be speculative of poor standing and are subject to very high credit risk.
Ca	Obligations rated Ca are highly speculative and are likely in, or very near, default, with some prospect of recovery of principal and interest.
C	Obligations rated C are the lowest rated and are typically in default, with little prospect for recovery of principal or interest.

Source: Moody’s Investors Service 2014:5

Table 2.2: Moody’s National scale ratings descriptions

National Long Term Rating Scale	
Aaa.n	Issuers or issues rated Aaa.n demonstrate the strongest creditworthiness relative to other domestic issuers.
Aa.n	Issuers or issues rated Aa.n demonstrate very strong creditworthiness relative to other domestic issuers.
A.n	Issuers or issues rated A.n present above-average creditworthiness relative to other domestic issuers.
Baa.n	Issuers or issues rated Baa.n represent average creditworthiness relative to other domestic issuers.
Ba.n	Issuers or issues rated Ba.n demonstrate below-average creditworthiness relative to other domestic issuers.
B.n	Issuers or issues rated B.n demonstrate weak creditworthiness relative to other domestic issuers.
Caa.n	Issuers or issues rated Caa.n demonstrate very weak creditworthiness relative to other domestic issuers.
Ca.n	Issuers or issues rated Ca.n demonstrate extremely weak creditworthiness relative to other domestic issuers.
C.n	Issuers or issues rated C.n demonstrate the weakest creditworthiness relative to other domestic issuers.

Source: Moody’s Investors Service 2014:12

2.6.2 Credit Risk Models

In this section a summary of the application of credit risk models in a South African context will be given. The same methodology will not be applied but the mentioned studies produced important results.

Smit, Swart and Van Niekerk (2003) applied the Merton (1974) model to South African data to test the model empirically in a South African context. Twenty investment grade companies were investigated with ratings AAA through BBB. The following equations were used for value of a risky bond $D(V,\tau)$, where V refers to the value of the firm as in Merton's model and follows the process $dV = \alpha_{(V)}Vdt + \sigma_{(V)}Vdz$ and B the face value of the bond.

The equity value of the firm follows the process $dF = \alpha_{(F)}Fdt + \sigma_{(F)}Fdz$ and because F is a function of V applying Itô's lemma yields the following

$$\sigma_{(F)}F = \sigma_{(V)}V\frac{\partial F(V,t)}{\partial V} \quad (2.20)$$

The resulting risk premium $R(\tau) - r$ is obtained by setting $D(V,\tau) = Be^{-R(\tau)\tau}$, where $R(\tau)$ is the yield-to-maturity of the bond.

$$D(V,\tau) = Be^{-r\tau}\left[N(h_2) + \frac{Ve^{r\tau}}{B}N(h_1)\right] \quad (2.21)$$

$$R(\tau) - r = -\frac{1}{\tau}\ln\left[N(h_2) + \frac{V(\tau)e^{r\tau}}{B}N(h_1)\right] \quad (2.22)$$

Where $h_2 = -\left[\frac{1}{2}\sigma_{(V)}^2\tau + \ln\left(\frac{Be^{-r\tau}}{V}\right)\right]/\sigma_{(V)}\sqrt{\tau}$

$$h_1 = -\sigma_{(V)}\sqrt{\tau} - h_2$$

The above equations are dependent on the firm's value $V(\tau)$ and its volatility $\sigma_{(V)}$, also the debt to be repaid B. The firm's market value is not easily obtained because the market value of debt is usually unobservable (Smit *et al*, 2003: 43). $F(\tau)$, the equity value of the firm is however observable and $\sigma_{(F)}$ can be estimated. This is then used to determine $V(\tau)$ and $\sigma_{(V)}$ in terms of $F(\tau)$, $\sigma_{(F)}$ and B using equation (2.20).

The probability of default can be inferred from equation (2.21) as follows, as well as the recovery rate, δ, in the event of default:

$$P(V(T) < B) = N(-h_2) \quad (2.23)$$

$$\delta = \frac{V(\tau)N(h_1)}{Be^{r\tau}N(-h_2)} \quad (2.24)$$

The actual debt of the companies was used in the analysis and financing of the firm's debt was observed at one year, three year and five year maturities.

Smit *et al* (2003) then also applied the Shimko, Tejima and van Deventer (STvD) model to the same twenty companies. The STvD model is a generalised Merton model that allows for stochastically varying interest rates. In this specific model it is assumed that short-term riskless interest rate follows a Vasicek process, $dr = k(\gamma - r)dt + \sigma_{(r)}d\tilde{z}$ with $dzd\tilde{z} = \rho dt$.

Under conditions of the StvD model the price of a risky bond and the value of the firm's debt can be obtained using the following equations:

$$P(\tau) = exp\left[\frac{1-e^{-k\tau}}{k}\left(\gamma - \frac{1}{2}\frac{\sigma_{(r)}^2}{k^2} - r\right) - \tau\left(\gamma - \frac{1}{2}\frac{\sigma_{(r)}^2}{k^2}\right) - \frac{\sigma_{(r)}^2}{4k^3}\left(1 - e^{-k\tau}\right)^2\right] \quad (2.25)$$

$$D(\tau) = V - N(h_1) + BP(\tau)N(h_2) \quad (2.26)$$

Where $h_1 = \frac{1}{\sqrt{\Sigma}}\left[\ln\frac{V}{P(\tau)B} + \frac{1}{2}\Sigma\right]$

$$h_2 = h_1 - \sqrt{\Sigma}$$

$$\Sigma = \tau\left(\sigma_{(V)}^2 + \frac{\sigma_{(r)}^2}{k^2} + \frac{2\rho\sigma_{(V)}\sigma_{(r)}}{k}\right) + \frac{e^{-k\tau}-1}{k^3}\left(2\sigma_{(r)}^2 + 2\rho\sigma_{(V)}\sigma_{(r)}k\right) - \frac{\sigma_{(r)}^2}{2k^3}\left(e^{-2k\tau} - 1\right)$$

Now considering the debt function $D(\tau) = Be^{-R(\tau)\tau}$ and $P(\tau) = e^{-r(\tau)\tau}$, the credit spread follows from substitution into equation (2.27):

$$R(\tau) - r(\tau) = -\frac{1}{\tau}ln\frac{PB}{D} \quad (2.27)$$

Table 2.3: Credit spreads obtained from Merton and Shimko models

Company	Rating	Volatility of assets	Leverage ratio	Credit Spread (basis points)							
				I year		3 year			5 year		
				Merton	Shimko	Merton	Shimko	Difference	Merton	Shimko	Difference
A	AAA	32%	25%	0,0	0,0	7	11	4	28	43	15
B*	AA+	19%	37%	0,0	0,0	1	3	2	6	15	9
C	AA	27%	24%	0,0	0,0	2	3	2	11	19	9
D*	AA	5%	84%	0,2	1,1	7	37	31	18	73	55
E*	AA	4%	87%	0,2	1,9	7	44	37	18	80	61
F*	AA	7%	82%	0,4	1,7	12	43	31	29	84	54
G*	AA	3%	91%	0,2	3,6	6	59	53	15	91	76
H	AA-	17%	41%	0,0	0,0	0	2	1	3	11	8
I	AA-	20%	46%	0,0	0,0	5	11	6	19	38	19
J	A+	21%	37%	0,0	0,0	2	4	2	10	21	11
K	A+	29%	38%	0,6	0,9	33	46	13	85	115	30
L*	A+	3%	93%	0,4	6,8	7	69	62	17	100	83
M*	A+	7%	84%	0,7	2,5	15	51	36	36	95	58
N*	A-	4%	89%	0,4	2,8	8	54	46	21	90	69
O	A-	24%	36%	0,0	0,0	6	11	5	24	41	17
P	A-	41%	13%	0,0	0,0	3	4	2	18	27	9
Q*	BBB+	30%	49%	16,0	19,1	160	191	31	297	346	49
R*	BBB	15%	69%	1,6	2,7	34	59	26	77	123	46
S*	BBB	18%	67%	7,4	10,3	84	116	33	169	220	51
T*	BBB	22%	60%	9,8	12,7	108	139	31	211	260	49

Source: Smit *et al* 2003:44

The asterisk indicates the firms that are from the banking sector. It was found that the spreads generated from the Merton (1974) model was significantly lower than the credit spreads in South African market. These empirical results were expected because the assumptions regarding interest rates could be seen as unrealistic and similar results were found by researchers in other countries when applying the model.

Using the STvD model with $\sigma_{(r)}$= 5% and ρ=30%, credit spreads were obtained that compare better with the credit spreads in the South African bond market. They found that the model showed a more realistic response to asset volatilities and leverage ratios. The credit spreads from the different models are compared in Table 2.3. The study validates the use of credit risk models for valuation purposes in South Africa.

Holman, *et al* (2011) investigated the Merton default probabilities' correlation with issued ratings to determine whether the Merton (1974) model could be used to quantify credit risk for South African firms. It was specifically referred to South Africa's history of default on listed debt that implies a firm's effective default probability is zero, which is not theoretically plausible (Holman *et al*, 2011: 2). The top 42 non-financial companies in the All Share Index were included in the analysis. Financial firms were not included because of the difference in capital structure and default point, which is the threshold that triggers default, in comparison with industrialised firms. Liabilities of financial firms decrease as the firm nears default and the opposite occurs for non-financial firms. The default point in Merton's model is the face value of the bond, but in this analysis it is estimated as the average of the firm's short and long term liabilities.

The firms are assumed to have a domestic benchmark financing rate, such as 3M JIBAR for all financial instruments. It is assumed that firms rated AAA are financed at the 3M JIBAR rate and firms rated below BBB+ are financed at prime. The 1 year forward swap rate on 15 June 2007, 9.62%, was used as forward indicator of the 3M JIBAR rate, this led to a spread of 3.38% and the historical average spread between 3M JIBAR and prime is 3.5%. These rates, which can be found in Table 2.4, were used as benchmark rates instead of the risk-free rate. There was also distinguished between base and worst case scenarios for the rates, with a 200 bps differential.

Table 2.4: Benchmark rates used in Holman *et al* (2011) study

Base case			Worst case = Base case + 2%		
Forward 9.62%	Spread 0.68%	Rate 10.30%	Forward 9.62%	Spread 2.68%	Rate 12.30%
AAA	0.68%	10.30%	AAA	2.68%	12.30%
AA	1.35%	10.97%	AA	3.35%	12.97%
A+	2.03%	11.65%	A+	4.03%	13.65%
A-	2.70%	12.32%	A-	4.70%	14.32%
BBB+	3.38%	13.00%	BBB+	5.38%	15.00%

Source: Holman *et al* 2011:7

Default probabilities were estimated over a time horizon of one year and market value of the firm's equity was calculated on 31 March 2007. The growth rate for the given firm's equity was calculated using the CAPM model and for the estimation of volatility the GARCH(1,1) model was implemented.

Default probabilities were then calculated for the base case and worst case scenarios and the firms were then ranked accordingly, with 1 being the most likely to default according the Merton model and 42 the least likely. These rankings were then compared to the ranking of the firms according to their published Moody's or Fitch ratings (Table 2.5). The credit ratings were assigned rating scores as follows, Aaa=20, Aa1=19, Aa2=18, and so forth ending with Ca=1. No apparent relationship between the published ratings and default probabilities obtained using the Merton (1974) model was found.

Two of the 42 companies for which the default probabilities were calculated were significantly higher than the rest of the companies and they were studied further in detail. They found that these two particular companies were highly leveraged and therefore the assets were closer to the default point relative to the other companies. This also highlights one of the flaws of the Merton model, which is that the security of outstanding debt is not taken into consideration and if a firm's capital structure has high use of leverage it does not necessarily imply an indication of default.

Table 2.5: Comparison of published rating rankings and Merton rankings

Rated Firm	Ticker	Rating	Rating Score (Moody's/Fitch)	Merton ranking (rated firms)	Merton ranking (all firms)
Anglogold	ANG	AA-	17	2	3
Barloworld	BAW	AA-	17	4	5
Bidvest	BVT	AA-	17	10	14
Pretoria Port. Cem.	PPC	AA-	17	16	35
BHP Billiton	BIL	A1	16	11	16
Steinhoff Int.	SHF	A	15	6	7
Anglo American plc	AGL	A2	15	15	34
Anglo Platinum	AMS	A2	15	19	41
Woolworths	WHL	A3	14	8	9
Telkom	TKG	A3	14	13	28
Imperial Holdings	IPL	Baa1	13	3	4
Sasol	SOL	Baa1	13	5	6
SA Breweries	SAB	Baa1	13	17	36
Harmony Gold	HAR	BBB	12	9	10
MTN Group	MTN	Baa3	11	7	8
Mittal Steel	MLA	Baa3	11	12	25
Sappi	SAP	Ba1	10	1	2
Lonmin plc	LON	Ba3	8	14	31
Edgars	EDC	B2	6	18	38

Source: Holman *et al* 2011:18

Low correlation between the default probabilities and ratings was found, $R^2 = 0.09$, and the results were contrary to what was expected, some of the higher rated firms exhibited higher default probabilities.

The fact that South African firms are not highly leveraged is considered as a reason for the overall low default probabilities produced by the Merton model, as financial leverage is one of the factors of calculation for the default probabilities.

It was concluded that the Merton (1974) model should only be used as a limited source of information regarding credit risk in South Africa.

2.7 SUMMARY

It can be seen that risk-neutral default probabilities are necessary for the quantification of CVA and there are various methods used for its estimation. The models can be separated into two categories, structural models and reduced form models. With structural models default is linked to the asset value of the firm or counterparty and Merton (1974) can be seen as the pioneer of these models. In reduced form models default is assumed to occur at any time and is not dependent on the firm or counterparty's asset value.

The term structure of credit spreads for investment grade bonds are generally upward sloping. Although credit spreads obtained from CDS data would give a more liquid, accurate perception of default likelihood, credit spreads can bond market data yields similar results and can be used for estimation of default probabilities. In many empirical studies only fixed-rate, non-callable bonds were used.

The Merton model has been applied to South African data and produced results which imply that credit risk models can be applied in the South African market and provides useful inference for credit risk. In the following chapter the methodology followed for obtaining default probabilities for different rating classes from South African data will be outlined.

CHAPTER 3
METHODOLOGY

3.1 INTRODUCTION

In this chapter methodology for estimating default probabilities using South African bond mark to market data will be given. The observed corporate bond credit spread data used for extrapolation to all unobservable maturities will be outlined. The methodology regarding the construction of the credit spread curves for different rating classes and calculation of the default probabilities using the obtained credit spreads will also be explained in detail, with reference to models and methods described in the previous chapter.

3.2 DATA

The month end bond mark to market data files for January 2014, February 2014 and March 2014 were obtained from the Johannesburg Stock Exchange (JSE) as well as a listing of all the bonds in the market on 8 April 2014. The credit ratings on the bonds, however, were outdated and ratings for the listed South African bonds had to be obtained from rating agencies such as Moody’s, Standard & Poor or Fitch. Credit ratings were then obtained from Moody’s of all the listed South African bonds which there were ratings available for. ISIN codes of the bonds in the Moody’s list of ratings were then used to identify and relate the ratings to the bonds in the JSE mark to market files.

Often in previous empirical research studies, as discussed in Chapter 2, only straight or plain vanilla bonds are considered and therefore bonds with callable options and floating rate coupons are excluded from this analysis. The excel file with all the listed South African bonds was filtered to obtain all the non-callable bonds with fixed coupons.

A secondary comma delimited excel file was then created containing the listed bonds with observed spread values and national scale long term ratings of three different rating classes: Aa.za, A.za and Baa.za. Global long term ratings were ignored as there was only one rating class, Baa.

3.3 CREDIT SPREAD MODELLING

The credit ratings from Moody’s were used for modelling the credit spreads for the different long term credit ratings to the unobservable maturities. The raw data described above was read into a statistical computer package, R (R Development Core Team, 2014), and different data sets were created to categorise the observations into the three observed national scale long term rating classes, Aa.za, A.za and Baa.za.

Plotting the observed spreads as a function of maturity per rating class, it was clear that there were outliers, in terms of what was expected for the rating classes. For certain ratings some of the observed spreads were higher than that of the lower credit ratings' spreads which is contrary to expectations; because lower credit ratings are expected to have higher credit spreads.

If the curves constructed from parameter estimation using non-linear regression analyses included all the observations it would lead to twisting curves for the different rating classes. Therefore the following rule was used for identifying outliers. Outliers were defined as spreads higher than the average spread of the rating just below the given rating and lower than the average spread of the rating just above the given rating. For example A.za spreads were bounded by the average Baa.za spreads above and the average Aa.za spreads below. This example is illustrated in Figure 3.1 the original A.za data points are displayed with the upper bound, Baa.za average and lower bound, Aa.za average.

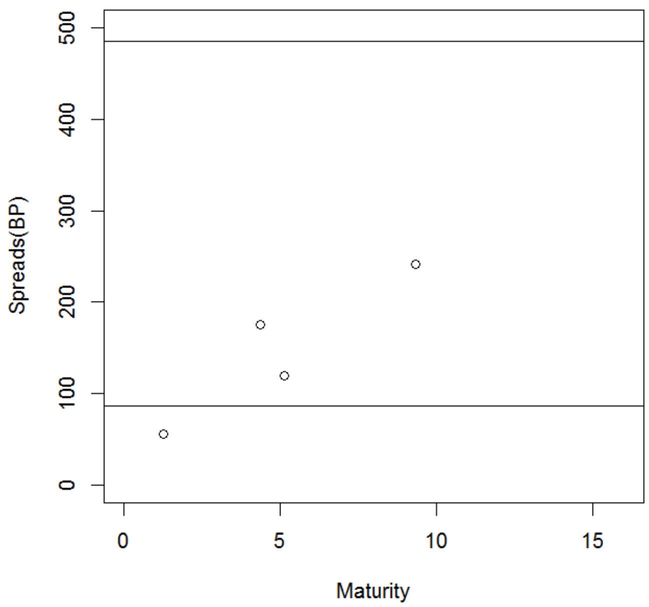

Figure 3.1: Plot of upper and lower bounds for A.za rating class

The outliers were then removed and if the data points left were insufficient to produce a regression model, data points within the range of the existing data points were added. Equation (2.18) was used for writing an R function that interpolates the spreads between the minimum and maximum maturities in the given data set and effectively adds two new interpolated observations.

The parametric curve (2.19) was used to execute the original non-linear regression analyses in R, but for some of the data sets the parameter estimates could not converge, which is sometimes a sign of over parameterisation and the curve was then simplified to the following:

$$s_t = a_1 + a_2 . \mathrm{e}^{a_3 \mathrm{t}} \qquad (3.1)$$

Where t denotes the time to maturity of the bond and s_t the basis point spread at t. For each of the rating classes a different set of optimised parameters $\{a_1, a_2, a_3\}$ were found and credit spreads for all unobservable maturities could now be extrapolated.

After filtering the listed bonds and removing the outliers there were not enough data points to construct curves differentiating the ranks within the given rating classes, that is, Aa1.za, Aa2.za and Aa3.za ratings within the Aa.za rating class. In fact, there were only enough data points for the modelling of the Aa.za and A.za curves and only one Baa.za observation.

According to the literature referred to in Chapter 2, the credit spread curves for investment grade rated bonds are all upward sloping with the lower credit quality spreads being higher than the higher credit quality spreads. For inference of the other curves, the curves that differ the assumption is made that the curves have the same overall slope and shape.

The curves obtained using the non-linear regression parameter estimates are then used to construct curves that are proportionate to each other; as outlined in the following six points:

i) The differences between the Aa.za and A.za curves and between the Baa.za and the A.za curves is averaged and used to construct the other curves using the Aa.za curve as the basis. Let D be defined as the above mentioned averaged difference.
ii) The Aa.za curve obtained from the non-linear regression is used as the basis curve. The new A.za curve is then obtained by adding D to the Aa.za curve and so also the new Baa.za curve is obtained by adding D to the new A.za curve.
iii) To differentiate the ranks within the rating class the above mentioned curves are assumed as the second ranking i.e. the Aa2.za, A2.za and Baa2.za curves.
iv) The space between these second ranked curves are then divided equally to fit in the third ranking of the higher rating and the first ranking of the lower rating. For example between the Aa2.za and A2.za curves would be the A1.za curve followed by the Aa3.za curve.
v) The Aa1.za curve is obtained by subtracting $\frac{D}{3}$ from the Aa2.za curve. The Baa3.za curve is obtained by adding $\frac{D}{3}$ to the Baa2.za curve.
vi) In the instance that the Aa2.za curve is too low to fit the Aa1.za curve, the Aa.za curve obtained from the regression analysis is used as the Aa1.za curve, thus the curves in (iii) are then the first ranking, in (iv) the second and third ranking of the higher rating are then

added and (v) adds the Baa2.za curve by adding $\frac{D}{3}$ to the Baa1.za curve and Baa3.za by adding $\frac{D}{3}$ to Baa2.za.

3.4 DEFAULT PROBABILTY CALCULATION

The following formula, (2.12) which was used by Hull *et al* (2004) and is specified for calculating risk-neutral default probabilities by Gregory (2012a: 205), is then used for calculating the cumulative default probability for each of the different ratings and rankings within the rating classes. The credit spreads calculated in the previous section is used for the calculation. Where default is governed by a Poisson process with hazard rate h that is approximated by $h \approx \frac{Spread}{1-R}$.

$$F(u) = 1 - \exp\left[-\frac{Spread}{1-R}u\right] \tag{3.2}$$

The justification and derivation of this formula can be found in Appendix B.

The marginal default probability which is necessary for CVA calculation, which is specified under Basel III, can then be obtained approximately by taking the difference between the cumulative default probabilities calculated from (3.2) at the specified times, it follows:

$$q(t_{i-1}, t_i) \approx \exp\left[-\frac{Spread_{t_{i-1}}}{1-R}t_{i-1}\right] - \exp\left[-\frac{Spread_{t_i}}{1-R}t_i\right] \tag{3.3}$$

3.5 SUMMARY

South African bond mark to market data was used to obtain risk neutral default probability estimates. Firstly credit spread curves were constructed by fitting a parametric curve to the observed credit spreads in the bond market. The parameters for the curves are obtained through execution of non-linear regression analyses and inferences are made to differentiate between rankings within the different rating classes. The corresponding cumulative default probabilities were then calculated for the rating classes. These credit spread curves and corresponding default probabilities for different rating classes and rankings, obtained from observed credit spreads from the South African bond market, can be used for CVA calculations.

CHAPTER 4
RESULTS

4.1 INTRODUCTION

This chapter contains the results obtained from applying the methodology described in Chapter 3 to construct credit spread curves for the different rating classes with differentiation between rankings within the rating classes. The credit spreads were then used for calculation of the default probability for different rating classes, also for different rankings in the rating classes.

4.2 CREDIT SPREADS

The parametric curves obtained from the output of the non-linear regression analysis and the curves constructed from it using the methodology discussed in Chapter 3 can be found in Figure 4.1. The plot on the left of Figure 4.1 is the curves constructed from the parameter estimates obtained from the non-linear regression analysis and the plot on the right is the proportionate curves obtained from using the differences and Aa.za curve as the basis curve, as discussed in Chapter 3.

After the filtering of the mark to market data there was only one Baa.za data point and the non-linear regression analysis was not executed. Therefore only Aa.za and A.za curves are displayed. When floating rate bonds are included there are more Baa.za rated bonds and Figure 4.3 shows the output where the Baa.za curve is displayed as well.

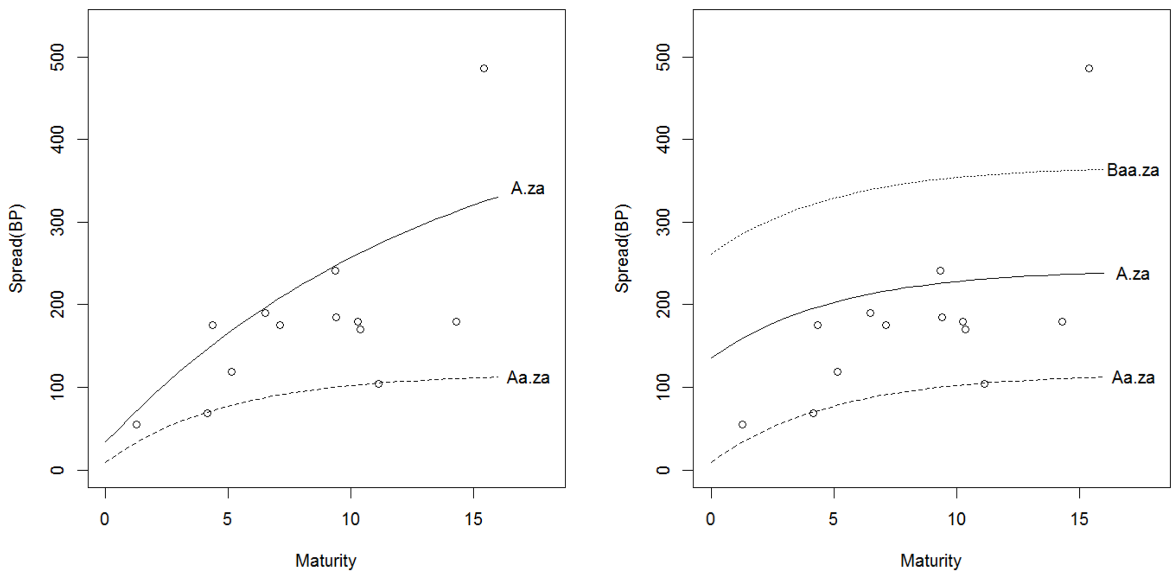

Figure 4.1: Plot of credit spreads for different rating classes

The curves that were obtained are in agreement with the literature on the term structure of investment grade credit spreads as discussed in Chapter 2. The curves are upward sloping and the higher ratings have flatter curves than the lower curves. The credit spreads also increase as the credit quality decreases.

In Figure 4.2 the different ranks within the given rating class is displayed, from The Aa1.za curve to the Baa3.za curve obtained from the methodology described in Chapter 3 and the rating is appended to the second ranked curve i.e. Aa2.za, A2.za, Baa2.za.

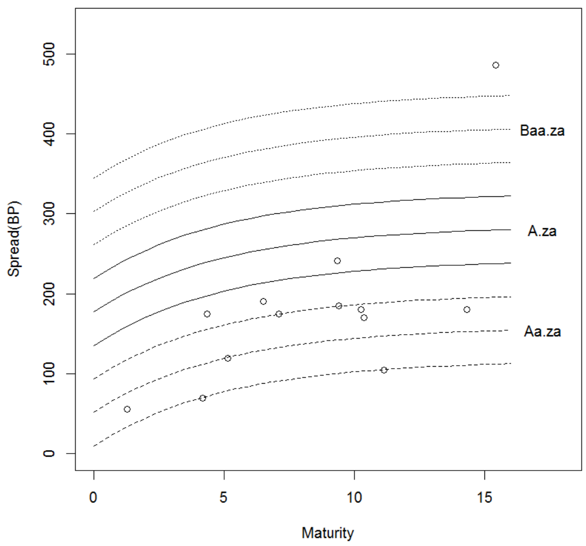

Figure 4.2: Credit spreads for different rankings within rating classes

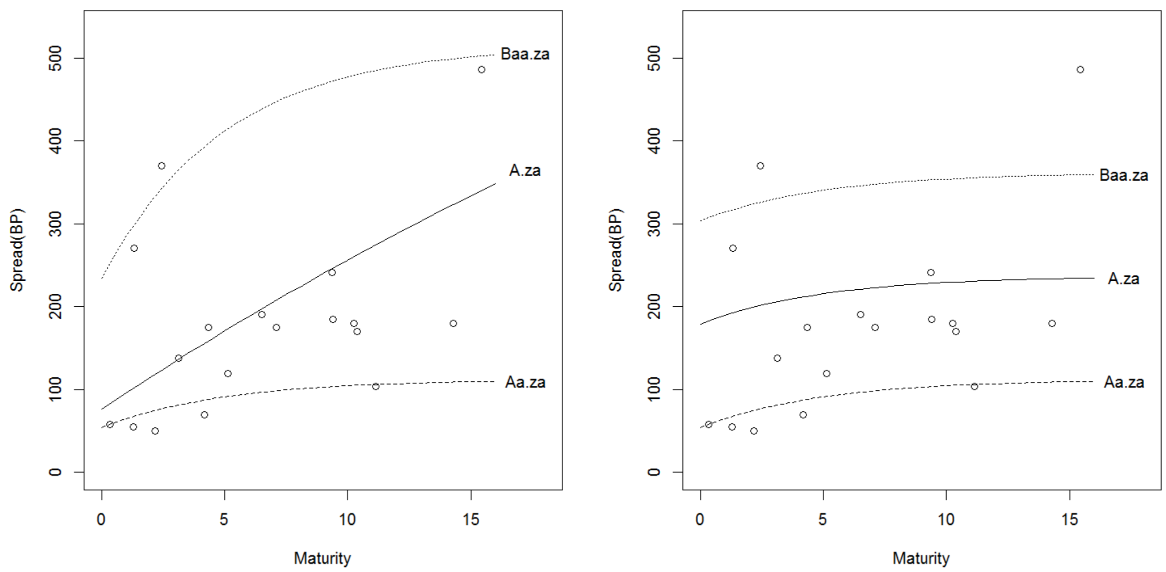

Figure 4.3: Credit spreads including floating rate bonds

4.3 DEFAULT PROBABILITIES

The cumulative default probabilities were calculated using equation (3.2). Figure 4.4 is a plot of the cumulative default probability calculated for the different ratings using the spreads displayed in Figure 4.2.

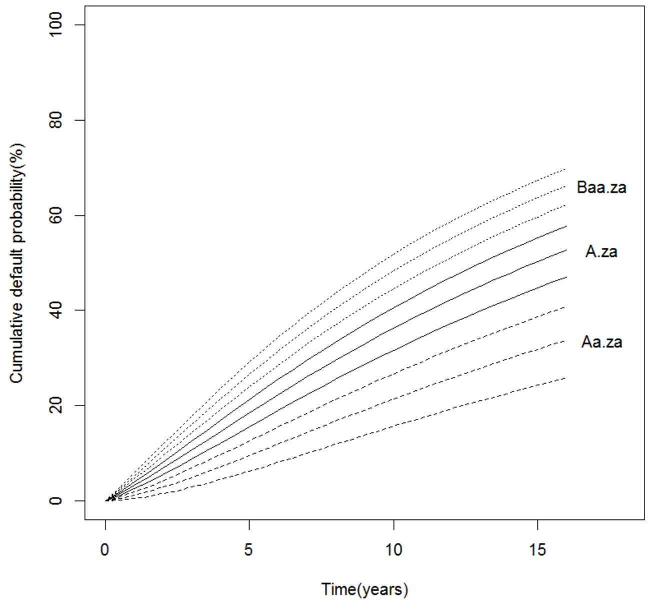

Figure 4.4: Cumulative default probabilities for different rating classes

The plot of Figure 4.4 agrees with the expectation that the cumulative default probability is an increasing function of time, because marginal default probabilities are increasing over time. The probability of default is also higher for lower credit ratings as the literature states that the likelihood of default increases as the credit quality of the counterparty decreases.

4.4 ROBUSTNESS TESTS

The code was developed from the bond mark to market data file of 31 January 2014. The code was executed on the 28 February 2014 and 31 March 2014 data files and delivered similar results. The February results are displayed in Figures 4.7 and 4.8 and the March results are displayed in Figure 4.9 and 4.10. These results are expected as there is not significant variation in the observed credit spreads over the month to month basis.

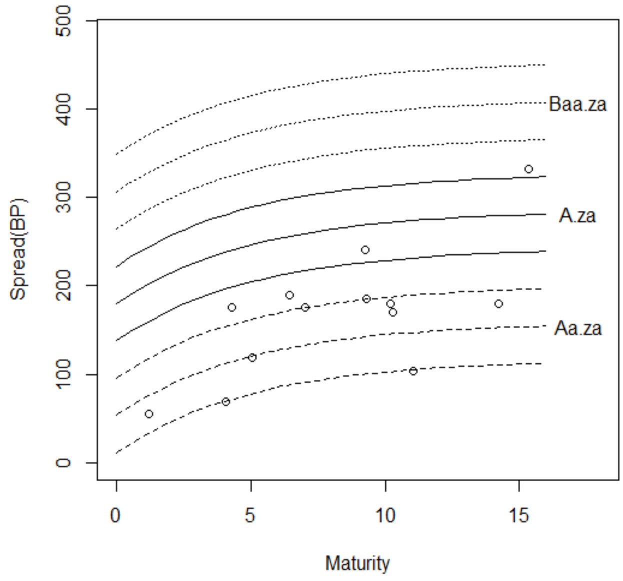

Figure 4.5: Spreads for different rating classes as on 28 February 2014

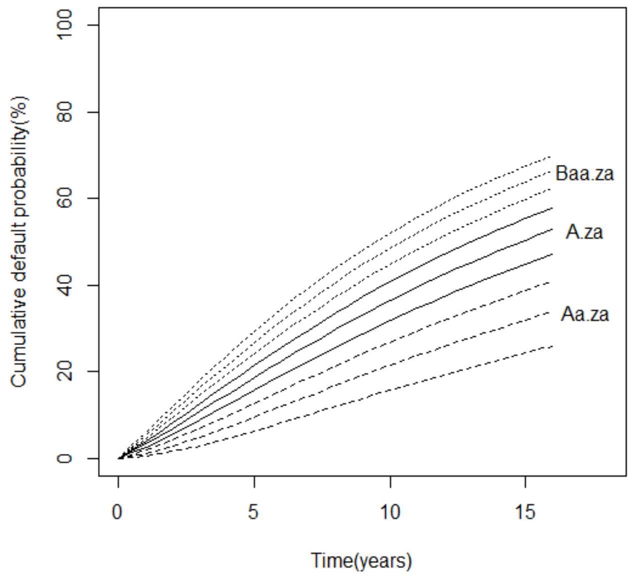

Figure 4.6: Cumulative default probabilities for different rating classes as on 28 February 2014

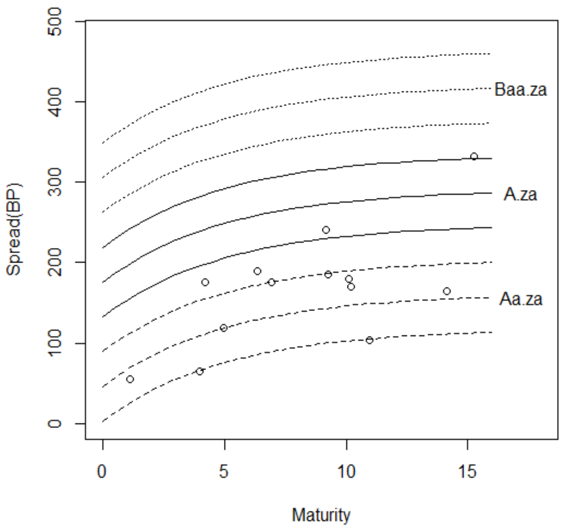

Figure 4.7: Cumulative default probabilities for different rating classes as on 31 March 2014

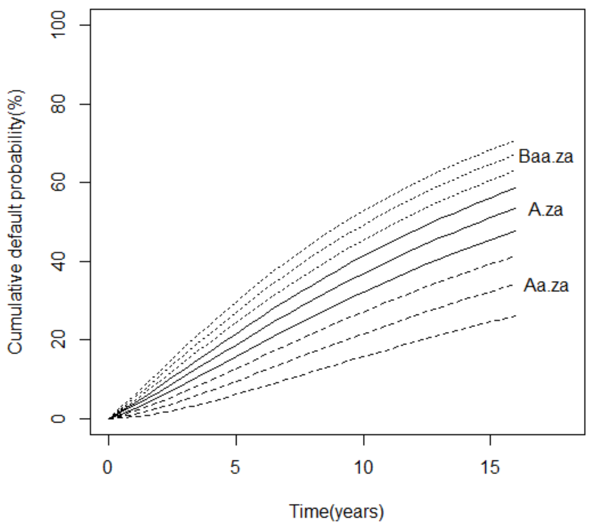

Figure 4.8: Cumulative default probabilities for different rating classes as on 31 March 2014

Table 4.1 shows the credit spreads for months January, February and March and in Table 4.2 the corresponding default probabilities are shown. There is not much variation in the credit spreads. The difference in the slope of the curves cause the February curves to be higher than the others in the beginning but as the maturity increases the spreads for March are higher.

Table 4.1: Credit Spreads for the three months' datasets

	Aa.za			A.za			Baa.za		
t	January	February	March	January	February	March	January	February	March
0	51.23517	53.20082	45.844	177.1627	179.8441	175.9139	303.0903	306.4874	305.9838
1	70.70551	72.36648	66.82447	196.6331	199.0098	196.8944	322.5606	325.653	326.9642
2	86.64648	88.058	84.00182	212.574	214.7013	214.0717	338.5016	341.3445	344.1416
3	99.69784	100.9051	98.06545	225.6254	227.5484	228.1353	351.553	354.1917	358.2052
4	110.3834	111.4235	109.5798	236.311	238.0667	239.6497	362.2385	364.71	369.7196
5	119.132	120.0352	119.0069	245.0595	246.6784	249.0768	370.9871	373.3217	379.1467
6	126.2947	127.0858	126.7252	252.2223	253.7291	256.7951	378.1499	380.3724	386.865
7	132.1591	132.8584	133.0444	258.0866	259.5017	263.1143	384.0142	386.1449	393.1842
8	136.9604	137.5846	138.2181	262.888	264.2279	268.288	388.8155	390.8711	398.3579
9	140.8914	141.4541	142.454	266.819	268.0973	272.5239	392.7465	394.7406	402.5938
10	144.1098	144.6221	145.922	270.0374	271.2654	275.9919	395.965	397.9087	406.0618
11	146.7448	147.2159	148.7614	272.6724	273.8592	278.8313	398.6	400.5025	408.9012
12	148.9022	149.3395	151.0861	274.8298	275.9828	281.156	400.7574	402.6261	411.2259
13	150.6685	151.0782	152.9894	276.5961	277.7215	283.0593	402.5237	404.3648	413.1292
14	152.1147	152.5017	154.5477	278.0422	279.145	284.6176	403.9698	405.7883	414.6875
15	153.2987	153.6672	155.8236	279.2262	280.3105	285.8935	405.1538	406.9537	415.9633
16	154.268	154.6214	156.8681	280.1956	281.2647	286.938	406.1232	407.9079	417.0079

Table 4.2: Default probabilities for the three months' datasets

	Aa.za			A.za			Baa.za		
t	January	February	March	January	February	March	January	February	March
0	0	0	0	0	0	0	0	0	0
1	1.171509	1.198864	1.107562	3.224099	3.262426	3.228313	5.234058	5.282888	5.303585
2	2.846906	2.892606	2.761222	6.840585	6.906618	6.88708	10.67009	10.75471	10.83788
3	4.862685	4.920097	4.785003	10.6682	10.75405	10.78024	16.11946	16.23005	16.39799
4	7.094648	7.159045	7.044861	14.57579	14.67572	14.76572	21.45452	21.58383	21.84528
5	9.450783	9.518909	9.441345	18.4714	18.58131	18.74388	26.59337	26.73604	27.09082
6	11.86449	11.93419	11.90243	22.2928	22.4098	22.64733	31.48722	31.63932	32.08172
7	14.28871	14.35861	14.37719	25.9997	26.12177	26.43249	36.11059	36.26921	36.79045
8	16.69102	16.76033	16.83061	29.56763	29.69335	30.07293	40.45398	40.61696	41.20679
9	19.04989	19.11818	19.2394	32.98331	33.11169	33.55435	44.51846	44.68416	45.33195
10	21.35161	21.41874	21.5888	36.24116	36.37152	36.87079	48.31185	48.47902	49.17438
11	23.58814	23.6541	23.87012	39.34083	39.47267	40.0219	51.84603	52.01369	52.74691
12	25.75535	25.82026	26.07893	42.28537	42.41831	43.01101	55.13511	55.30248	56.06469
13	27.85185	27.91586	28.21374	45.07994	45.21369	45.84367	58.19418	58.36061	59.14387
14	29.87804	29.94134	30.27501	47.73092	47.86524	48.52674	61.0385	61.20347	62.00077
15	31.83552	31.8983	32.26444	50.24531	50.38	51.06776	63.68301	63.84606	64.65129
16	33.72659	33.78901	34.18451	52.63034	52.76519	53.47443	66.14201	66.30277	67.11064

4.5 COMPARISON TO SOUTH AFRICAN RESULTS

The studies conducted by Smit *et al* (2003), on 20 South African companies and Holman *et al* (2011) on the top 42 non-financial South African companies, produced similar when applying Merton's model. Both studies reported credit spreads close to zero, especially for short maturities, which were lower than the actual credit spreads observed in the market. Smit *et al* (2003) also implemented the STvD model which produced more realistic spreads, which can be seen in Table 2.3. Table 4.3 presents a comparison of the credit spreads obtained from the methodology outlined in Chapter 3 and the average 5 year credit spreads for the different rated companies obtained by Smit *et al* (2003) applying the STvD model. The methodology followed in this paper resulted in higher spreads, although these values are not completely comparable because the ratings used in this paper are national scale ratings and not global ratings which were used by Smit *et al* (2003) and there are other factors, discussed in Chapter 2, which also affect credit spreads and can vary over time.

Table 4.3: Comparison to Smit *et al* (2003) credit spreads

		Smit *et al* (2003) applying STvD model	
Rating	5 year credit spread(BP)	Rating	5 year average credit spread(BP)
Aa.za	119.132	AA	51.38
A.za	245.0595	A	69.86
Baa.za	370.9871	BBB	237.25

4.6 CONCLUSION

Overall, the resulting credit spreads and default probabilities yield comparable predictions that can be used for CVA calculation. The model relies heavily on the use of observed credit spreads from the bond mark to market data and is therefore representative of the market. The method used for default probability calculation using the credit spreads is also directly in line with the intended use of the default probabilities, which is CVA quantification.

In the following chapter the research will be viewed as a whole; overall conclusions will be drawn and final remarks will be made. The research will be assessed in terms of specifying where there is room for improvement and possibilities for further study will be stated.

CHAPTER 5
CONCLUSION

Credit risk and the management thereof, is still a developing field. There is an increasing importance of incorporating counterparty credit risk (CCR) into derivative valuation as well as financial reporting in the form of CVA. Default probabilities are necessary for the quantification of CVA and there are many different models and methods for estimating default probabilities.

The relevant literature regarding default probabilities was reviewed in detail. It was found that default probabilities are necessary for credit risk quantification in the form of credit valuation adjustments (CVA) and there are various models and methods for estimating risk-neutral default probabilities and credit spreads.

The relationship between corporate bond spreads and credit default swap spreads makes it is possible to use bond mark to market data for the estimation of default probabilities and ultimately the quantification of CVA. The implementation of credit risk models on South African data delivered important results and showed that models such as Merton (1974) could be used for quantification of credit risk in a South African context.

The methodology followed for obtaining credit spreads for the different rating classes and all unobservable maturities from South African bond mark to market data was given in Chapter 3. The credit spreads for the different rating classes were used for estimation of the default probability for the different rating classes.

The results obtained in Chapter 4 showed that the method can be used for different sets of data and is not just applicable to the original data set it was developed from. Comparing the results with previous results from studies on the South African market showed that the model delivers results which are in line with expectations and delivers predictions that can be used for the purpose of CVA quantification.

The limited availability of the ratings data and the spreads listed in the bond mark to market files might have led to larger bias in the estimates obtained from the model, because the method of estimation relies heavily on the data. Therefore another data source other than JSE might have to be considered. The model is also not completely dynamic, because of the preparation of the data beforehand with the filtering and integration of the ratings. Certain assumptions were also made and it is therefore not completely generalised. Also, speculative grade ratings were not in the observed data set, but it is also necessary to calculate the default probability of such rated counterparties.

A suggestion for further research, also using bond market data to calculate default probabilities, could be considering a reduced form model, possibly the model specified by Hull & White

(2000). This model would still use actual bond market data but not the given spreads; it would calculate the present value of the loss incurred in the event of a default and then use equation (2.9) to infer the default probability.

Overall it can be deduced that South African bond mark to market data can be used for inferring default probabilities and ultimately used for the calculation of CVA.

APPENDIX A

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APPENDIX B
CUMULATIVE DEFAULT PROBABILITY

If default is assumed to be driven by a Poisson process with constant hazard rate h, the cumulative default probability is given by:

$$F(u) = 1 - \exp[-hu] \quad \text{(A.1)}$$

The instantaneous default probability which is implied from A1 is:

$$\frac{dF(u)}{du} = h\exp[-hu] \quad \text{(A.2)}$$

Because $exp[-hu]$ denotes the survival probability until time u, h can be interpreted as the instantaneous forward default probability.

The hazard rate can be linked to the CDS spread under the assumption that all cashflows are continuous. The value of the cashflows follows as

$$\int_0^T B(u)S(u)du \quad \text{(A.3)}$$

Where $B(u)$ denotes the risk-free discount factor and $S(u)$ the survival probability which can also be written as $1 - F(u)$.

The CDS protection value can be represented as:

$$(1-R)\int_0^T B(u)dF(u) = (1-R)h\int_0^T B(u)S(u)du \quad \text{(A.4)}$$

The CDS spread can be seen as the unit cost of the protection, which is A.4 divided by A.3, therefore:

$$Spread = (1-R)h \text{ or } h = \frac{Spread}{1-R} \quad \text{(A.5)}$$

APPENDIX C
R CODE

```
#creates data frame with non-callable fixed rate bonds
bonddata<-read.table("E:\\Research\\Besa\\jan.csv",sep=",",header=TRUE)
#creates data frame with non-callable bonds
bonddata<-read.table("E:\\Research\\Besa\\jan2.csv",sep=",",header=TRUE)

#creating data frames for the different ratings
Aa.za<-
bonddata[bonddata[,"Rating"]=="Aa1.za"|bonddata[,"Rating"]=="Aa2.za"|bonddata
[,"Rating"]=="Aa3.za",]
A.za<-
bonddata[bonddata[,"Rating"]=="A1.za"|bonddata[,"Rating"]=="A2.za"|bonddata[,
"Rating"]=="A3.za",]
Baa.za<-
bonddata[bonddata[,"Rating"]=="Baa1.za"|bonddata[,"Rating"]=="Baa2.za"|bondda
ta[,"Rating"]=="Baa3.za",]

#removing outliers, outliers defined as spreads higher than the average
spread of the rating just below the given rating and lower than the average
spread of the rating just above the given rating

Baa.za_ave<-mean(Baa.za[,7])
A.za<-A.za[A.za[,7]<Baa.za_ave,]
A.za_ave<-mean(A.za[,7])
Aa.za<-Aa.za[Aa.za[,7]<A.za_ave,]

Aa.za_ave<-mean(Aa.za[,7])
A.za<-A.za[A.za[,7]>Aa.za_ave,]
A.za_ave<-mean(A.za[,7])
Baa.za<-Baa.za[Baa.za[,7]>A.za_ave,]

#function that adds 3 interpolated points to the dataset
mod<-function(A)
{
#interpolated points maturities
tau1<-min(A[,4])+(max(A[,4])-min(A[,4]))/3
tau2<-max(A[,4])-(max(A[,4])-min(A[,4]))/3

t1<-min(A[,4])
r1<-A[A[,4]==min(A[,4]),7]/100
```

```
t2<-max(A[,4])
r2<-A[A[,4]==max(A[,4]),7]/100

#interpolated spreads
rtau1<-((tau1-t1)/(t2-t1))*(t2/tau1)*r2 + ((t2-tau1)/(t2-t1))*(t1/tau1)*r1
rtau2<-((tau2-t1)/(t2-t1))*(t2/tau2)*r2 + ((t2-tau2)/(t2-t1))*(t1/tau2)*r1

#new rows added to the dataset
int1<-data.frame(Bond.Code=NA, ISIN.Code=NA, Maturity.Date=NA, Maturity=tau1,
Coupon=NA, Companion.Bond=NA, BP.Spread=rtau1*100, Rating=NA, Trade.date=NA)
int2<-data.frame(Bond.Code=NA, ISIN.Code=NA, Maturity.Date=NA, Maturity=tau2,
Coupon=NA, Companion.Bond=NA, BP.Spread=rtau2*100, Rating=NA, Trade.date=NA)
return(A=rbind(A,int1,int2))
}

#new datasets used for regression analysis
A.zamod<-mod(A.za)
Aa.zamod<-mod(Aa.za)
Baa.zamod<-mod(Baa.za)

#regression analysis fitting the curve spread=a1+a2*exp(a3*maturity) for
Aa.za rating
startvals<-list(a1=160,a2=-90,a3=-0.2)
Aa.zamodel<-
nls(BP.Spread~a1+a2*exp(a3*Maturity),data=Aa.zamod,start=startvals,trace=TRUE
,algorithm="port",lower=c(0,-1000,-0.2))
a1<-coef(Aa.zamodel)[1]
a2<-coef(Aa.zamodel)[2]
a3<-coef(Aa.zamodel)[3]

#regression analysis fitting the curve spread=b1+b2*exp(b3*maturity) for A.za
rating
startvals<-list(b1=220,b2=-110,b3=-0.2)
A.zamodel<-
nls(BP.Spread~b1+b2*exp(b3*Maturity),data=A.zamod,start=startvals,trace=TRUE,
algorithm="port",lower=c(0,-1000,-0.2))
b1<-coef(A.zamodel)[1]
b2<-coef(A.zamodel)[2]
b3<-coef(A.zamodel)[3]

#regression analysis fitting the curve spread=c1+c2*exp(c3*maturity) for
Baa.za rating
```

```
startvals<-list(c1=250,c2=-150,c3=-0.2)
Baa.zamodel<-
nls(BP.Spread~c1+c2*exp(c3*Maturity),data=Baa.zamod,start=startvals,trace=TRU
E,algorithm="port",lower=c(0,-1000,-0.2))
c1<-coef(Baa.zamodel)[1]
c2<-coef(Baa.zamodel)[2]
c3<-coef(Baa.zamodel)[3]

#plot of original data points
par(mfrow=c(1,2))
plot(bonddata[,4],bonddata[,7],xlab="Maturity",ylab="Spread(BP)",xlim=c(0,flo
or(max(na.omit(bonddata[,4])))+3),ylim=c(0,max(na.omit(bonddata[,7])+50)))
t<-seq(0,floor(max(na.omit(bonddata[,4])))+1,by=0.5)

#plotting the fitted curves
lines(t,a1+a2*exp(a3*t),lty=2)
lines(t,b1+b2*exp(b3*t),lty=1)
lines(t,c1+c2*exp(c3*t),lty=3)

text(x=17.25,y=a1+a2*exp(a3*17.25),labels="Aa.za")
text(x=17.25,y=b1+b2*exp(b3*17.25),labels="A.za")
text(x=17.25,y=c1+c2*exp(c3*17.25),labels="Baa.za")

diff1<-mean((b1+b2*exp(b3*t))-(a1+a2*exp(a3*t)))
diff2<-mean((c1+c2*exp(c3*t))-(b1+b2*exp(b3*t)))

diff<-mean(na.omit(diff1,diff2))

plot(bonddata[,4],bonddata[,7],xlab="Maturity",ylab="Spread(BP)",xlim=c(0,flo
or(max(na.omit(bonddata[,4])))+3),ylim=c(0,max(na.omit(bonddata[,7])+50)))
lines(t,a1+a2*exp(a3*t),lty=2)
lines(t,a1+a2*exp(a3*t)+diff,lty=1)
lines(t,a1+a2*exp(a3*t)+2*diff,lty=3)

text(x=17.25,y=a1+a2*exp(a3*17.25),labels="Aa.za")
text(x=17.25,y=a1+a2*exp(a3*17.25)+diff,labels="A.za")
text(x=17.25,y=a1+a2*exp(a3*17.25)+2*diff,labels="Baa.za")

windows()
plot(bonddata[,4],bonddata[,7],xlab="Maturity",ylab="Spread(BP)",xlim=c(0,flo
or(max(na.omit(bonddata[,4])))+3),ylim=c(0,max(na.omit(bonddata[,7])+50)))
```

```
Aa1.za<-(a1+a2*exp(a3*t))-ifelse((a1+a2)>diff/3,diff/3,0)
Aa2.za<-Aa1.za+diff/3
Aa3.za<-Aa2.za+diff/3

lines(t,Aa1.za,lty=2)
lines(t,Aa2.za,lty=2)
lines(t,Aa3.za,lty=2)

A1.za<-Aa3.za+diff/3
A2.za<-A1.za+diff/3
A3.za<-A2.za+diff/3

lines(t,A1.za,lty=1)
lines(t,A2.za,lty=1)
lines(t,A3.za,lty=1)

Baa1.za<-A3.za+diff/3
Baa2.za<-Baa1.za+diff/3
Baa3.za<-Baa2.za+diff/3

lines(t,Baa1.za,lty=3)
lines(t,Baa2.za,lty=3)
lines(t,Baa3.za,lty=3)

text(x=17.25,y=c(max(Aa2.za),max(A2.za),max(Baa2.za)),labels=c("Aa.za","A.za"
,"Baa.za"))

#PDs calulated for t time points
Aa1.zaPD=(1-exp(-t*Aa1.za/(0.6*10000)))*100
Aa2.zaPD=(1-exp(-t*Aa2.za/(0.6*10000)))*100
Aa3.zaPD=(1-exp(-t*Aa3.za/(0.6*10000)))*100

A1.zaPD=(1-exp(-t*A1.za/(0.6*10000)))*100
A2.zaPD=(1-exp(-t*A2.za/(0.6*10000)))*100
A3.zaPD=(1-exp(-t*A3.za/(0.6*10000)))*100

Baa1.zaPD=(1-exp(-t*Baa1.za/(0.6*10000)))*100
Baa2.zaPD=(1-exp(-t*Baa2.za/(0.6*10000)))*100
Baa3.zaPD=(1-exp(-t*Baa3.za/(0.6*10000)))*100

#table of spreads
```

```
Spreads<-data.frame(t=t, Aa1.za=Aa1.za, Aa2.za=Aa2.za, Aa3.za=Aa3.za,
A1.za=A1.za, A2.za=A2.za, A3.za=A3.za, Baa1.za=Baa1.za, Baa2.za=Baa2.za,
Baa3.za=Baa3.za)

#table of PDs
PDs<-data.frame(t=t, Aa1.za=Aa1.zaPD, Aa2.za=Aa2.zaPD, Aa3.za=Aa3.zaPD,
A1.za=A1.zaPD, A2.za=A2.zaPD, A3.za=A3.zaPD, Baa1.za=Baa1.zaPD,
Baa2.za=Baa2.zaPD, Baa3.za=Baa3.zaPD)

#plot of cumulative defalut probability
windows()
plot(t,Aa1.zaPD,ylim=c(0,100),xlab="Time(years)",ylab="Cumulative default
probability(%)",type="l",lty=2,xlim=c(0,18))
lines(t,Aa2.zaPD,lty=2)
lines(t,Aa3.zaPD,lty=2)
lines(t,A1.zaPD,lty=1)
lines(t,A2.zaPD,lty=1)
lines(t,A3.zaPD,lty=1)
lines(t,Baa1.zaPD,lty=3)
lines(t,Baa2.zaPD,lty=3)
lines(t,Baa3.zaPD,lty=3)

text(x=17.25,y=c(max(Aa2.zaPD),max(A2.zaPD),max(Baa2.zaPD)),labels=c("Aa.za",
"A.za","Baa.za"))


#creating tables of spreads and default probabilities in excel
write.csv(Spreads, file = "E:\\Research\\Besa\\non-callable
Spreads.csv",row.names=FALSE)

write.csv(PDs, file = "E:\\Research\\Besa\\non-callable
PDs.csv",row.names=FALSE)

write.csv(Spreads, file = "E:\\Research\\Besa\\non-callable fixed
Spreads.csv",row.names=FALSE)

write.csv(PDs, file = "E:\\Research\\Besa\\non-callable fixed
PDs.csv",row.names=FALSE)
```

APPENDIX D

TABLES OF SPREADS AND DEFAULT PROBABILITIES

Table C.1: Spreads of non-callable fixed rate bonds

t	Aa1.za	Aa2.za	Aa3.za	A1.za	A2.za	A3.za	Baa1.za	Baa2.za	Baa3.za
0	9.259318	51.23517	93.21103	135.1869	177.1627	219.1386	261.1144	303.0903	345.0662
1	28.72966	70.70551	112.6814	154.6572	196.6331	238.6089	280.5848	322.5606	364.5365
2	44.67063	86.64648	128.6223	170.5982	212.574	254.5499	296.5258	338.5016	380.4775
3	57.72199	99.69784	141.6737	183.6496	225.6254	267.6013	309.5771	351.553	393.5288
4	68.40754	110.3834	152.3592	194.3351	236.311	278.2868	320.2627	362.2385	404.2144
5	77.15613	119.132	161.1078	203.0837	245.0595	287.0354	329.0113	370.9871	412.963
6	84.31886	126.2947	168.2706	210.2464	252.2223	294.1981	336.174	378.1499	420.1257
7	90.18322	132.1591	174.1349	216.1108	258.0866	300.0625	342.0383	384.0142	425.9901
8	94.98455	136.9604	178.9363	220.9121	262.888	304.8638	346.8397	388.8155	430.7914
9	98.91554	140.8914	182.8672	224.8431	266.819	308.7948	350.7707	392.7465	434.7224
10	102.134	144.1098	186.0857	228.0615	270.0374	312.0132	353.9891	395.965	437.9408
11	104.769	146.7448	188.7207	230.6966	272.6724	314.6483	356.6241	398.6	440.5758
12	106.9264	148.9022	190.8781	232.8539	274.8298	316.8056	358.7815	400.7574	442.7332
13	108.6927	150.6685	192.6444	234.6202	276.5961	318.572	360.5478	402.5237	444.4995
14	110.1388	152.1147	194.0905	236.0664	278.0422	320.0181	361.9939	403.9698	445.9456
15	111.3228	153.2987	195.2745	237.2504	279.2262	321.2021	363.1779	405.1538	447.1296
16	112.2922	154.268	196.2439	238.2197	280.1956	322.1714	364.1473	406.1232	448.099

Table C.2: Default probabilities of non-callable fixed rate bonds

t	Aa1.za	Aa2.za	Aa3.za	A1.za	A2.za	A3.za	Baa1.za	Baa2.za	Baa3.za
0	0	0	0	0	0	0	0	0	0
1	0.477683	1.171509	1.860498	2.544683	3.224099	3.898778	4.568754	5.234058	5.894725
2	1.47799	2.846906	4.196801	5.527941	6.840585	8.13499	9.41141	10.67009	11.91129
3	2.844849	4.862685	6.838613	8.773501	10.6682	12.52356	14.34037	16.11946	17.86159
4	4.458075	7.094648	9.658462	12.15153	14.57579	16.93316	19.22547	21.45452	23.62206
5	6.227333	9.450783	12.56343	15.56907	18.4714	21.27396	23.98018	26.59337	29.11673
6	8.086187	11.86449	15.48749	18.96155	22.2928	25.48712	28.55012	31.48722	34.30358
7	9.986791	14.28871	18.38502	22.28557	25.9997	29.53633	32.90393	36.11059	39.164
8	11.89545	16.69102	21.22557	25.51329	29.56763	33.4013	37.02629	40.45398	43.69509
9	13.78908	19.04989	23.98967	28.62801	32.98331	37.07283	40.91281	44.51846	47.90408
10	15.65235	21.35161	26.66578	31.62087	36.24116	40.54926	44.56626	48.31185	51.80435
11	17.47562	23.58814	29.2479	34.48845	39.34083	43.83381	47.99399	51.84603	55.41276
12	19.25327	25.75535	31.73385	37.23094	42.28537	46.93281	51.20601	55.13511	58.74783
13	20.9825	27.85185	34.12401	39.85091	45.07994	49.85439	54.21378	58.19418	61.82855
14	22.66245	29.87804	36.42042	42.3524	47.73092	52.60763	57.02934	61.0385	64.67361
15	24.29356	31.83552	38.62615	44.74028	50.24531	55.20193	59.66477	63.68301	67.30095
16	25.87711	33.72659	40.74482	47.01984	52.63034	57.6467	62.13184	66.14201	69.72751

Table C.3: Spreads including floating rate bonds

t	Aa1.za	Aa2.za	Aa3.za	A1.za	A2.za	A3.za	Baa1.za	Baa2.za	Baa3.za
0	12.46204	54.05527	95.6485	137.2417	178.835	220.4282	262.0214	303.6146	345.2079
1	22.99346	64.58669	106.1799	147.7731	189.3664	230.9596	272.5528	314.146	355.7393
2	31.61585	73.20908	114.8023	156.3955	197.9888	239.582	281.1752	322.7684	364.3617
3	38.67527	80.2685	121.8617	163.455	205.0482	246.6414	288.2346	329.8279	371.4211
4	44.45504	86.04826	127.6415	169.2347	210.8279	252.4212	294.0144	335.6076	377.2009
5	49.18711	90.78033	132.3736	173.9668	215.56	257.1532	298.7465	340.3397	381.9329
6	53.0614	94.65462	136.2479	177.8411	219.4343	261.0275	302.6208	344.214	385.8072
7	56.2334	97.82663	139.4199	181.0131	222.6063	264.1995	305.7928	347.386	388.9792
8	58.83041	100.4236	142.0169	183.6101	225.2033	266.7966	308.3898	349.983	391.5762
9	60.95667	102.5499	144.1431	185.7364	227.3296	268.9228	310.516	352.1093	393.7025
10	62.6975	104.2907	145.884	187.4772	229.0704	270.6636	312.2569	353.8501	395.4433
11	64.12277	105.716	147.3092	188.9025	230.4957	272.0889	313.6821	355.2754	396.8686
12	65.28969	106.8829	148.4761	190.0694	231.6626	273.2558	314.8491	356.4423	398.0355
13	66.24508	107.8383	149.4315	191.0248	232.618	274.2112	315.8044	357.3977	398.9909
14	67.02728	108.6205	150.2137	191.807	233.4002	274.9934	316.5866	358.1799	399.7731
15	67.6677	109.2609	150.8542	192.4474	234.0406	275.6338	317.2271	358.8203	400.4135
16	68.19203	109.7853	151.3785	192.9717	234.5649	276.1582	317.7514	359.3446	400.9378

Table C.4: Default probabilities including floating rate bonds

t	Aa1.za	Aa2.za	Aa3.za	A1.za	A2.za	A3.za	Baa1.za	Baa2.za	Baa3.za
0	0	0	0	0	0	0	0	0	0
1	0.382491	1.070672	1.754099	2.432804	3.106821	3.776182	4.440918	5.101062	5.756646
2	1.048328	2.410768	3.754449	5.079629	6.386563	7.675502	8.946694	10.20038	11.43681
3	1.915186	3.933954	5.911171	7.847694	9.744359	11.60199	13.42138	15.20333	16.9486
4	2.920183	5.575112	8.157435	10.66914	13.11215	15.48835	17.79957	20.04758	22.23411
5	4.016056	7.285961	10.44447	13.49538	16.44235	19.28892	22.03852	24.69445	27.2599
6	5.167821	9.031294	12.73737	16.29246	19.70271	22.97403	26.11207	29.12228	32.00984
7	6.349988	10.7859	15.01169	19.03732	22.87227	26.52557	30.00583	33.32124	36.4796
8	7.544298	12.53209	17.2508	21.71494	25.93826	29.93373	33.71366	37.28967	40.67276
9	8.737937	14.25778	19.44377	24.31608	28.89371	33.19446	37.23509	41.03133	44.59796
10	9.922145	15.95501	21.58382	26.83565	31.73575	36.30767	40.57339	44.55342	48.26689
11	11.09114	17.61878	23.66716	29.27147	34.46432	39.27591	43.73424	47.86523	51.69294
12	12.24132	19.24625	25.69205	31.62335	37.0812	42.10341	46.72474	50.97719	54.89022
13	13.37061	20.83609	27.65822	33.89243	39.5894	44.79542	49.5528	53.9002	57.87296
14	14.47804	22.38798	29.56633	36.08075	41.99265	47.35776	52.22665	56.64521	60.6551
15	15.5634	23.90231	31.41768	38.19083	44.29507	49.79646	54.75453	59.22295	63.25007
16	16.62698	25.37993	33.21395	40.22551	46.50096	52.11758	57.14454	61.64374	65.67058