A structural approach to assessing default probabilities of South African public companies

Report presented in partial fulfilment
of the requirements for the degree of
BCommHons **(Financial Risk Management)**
at the University of Stellenbosch

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Degree of confidentiality: A

October 2016

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Acknowledgements

The author wishes to acknowledge the Department of Statistics and Actuarial Science (University of Stellenbosch) for generously making available their research report template and guidelines.

The author wishes to acknowledge the tireless work and patience of Mr. Carel Van Der Merwe in his efforts and contributions towards the development of this research.

Abstract

Following the 2008 financial crisis, global regulatory authorities have highlighted the need for transparency in over the counter derivative transactions as well as the quantification of counterparty credit risk in the form of credit value adjustments (CVA) amongst others. Default probabilities are essential to multiple facets of the measurement and management of credit risk and are essential for CVA, which are required by the International Financial Reporting Standards. Default probabilities can be determined from quoted bond prices in the markets or credit default swap (CDS) spreads. However, bond and CDS market data are not always available and may be particularly complex for the counterparty being evaluated in the transaction. Structural models apply an option-theoretic approach inspired by Merton (1974) that uses equity market and financial statement data in order to determine default probabilities. The research found that the Merton and Delianedis & Geske (D&G) structural models provide limited information regarding the credit risk of firms in the South African market. The low levels of leverage amongst South African firms was found to be a primary reason for the inability of the basic structural models to capture the credit risks associated with the firms. Moreover, the extensions of the Merton (1974) model although practically challenging to implement, may provide a consistent and reliable manner in which to determine default probabilities from financial statement and equity market data.

Key words:

Credit valuation adjustments; Default probabilities; Structural models of default probabilities; South African equity market, South African financial statement information; Credit default swaps

Opsomming

Na aanleiding van die finansiële krisis van 2008, het die globale regulerende owerhede die behoefte beklemtoon vir deursigtigheid in oor die toonbank finansiële afgeleide instrument transaksies, asook die kwantifisering van teenparty kredietrisiko in die vorm van kredietwaarde-aanpassings (CVA), onder andere. Wanbetaling waarskynlikhede is noodsaaklik om verskeie fasette van die meting en bestuur van kredietrisiko en is noodsaaklik vir CVA, wat vereis word deur die Internasionale Finansiële Verslagdoeningstandaarde. Wanbetaling waarskynlikhede kan bepaal word vanaf gekwoteerde verbande in die mark of Crediet wanbetaling uitruilkontrakte (CDS). Maar verbande en CDS mark data is nie altyd beskikbaar nie, en kan veral kompleks raak vir die geëvalueerde teenparty in die transaksie. Strukturele modelle pas 'n opsie-teoretiese benadering geïnspireer deur Merton (1974) wat die aandelemark en die finansiële state data gebruik om die wanbetaling waarskynlikhede te bepaal. Die navorsing het bevind dat die Merton en Delianedis & Geske (D&G) strukturele modelle bied beperkte inligting oor die kredietrisiko van maatskappye in die Suid-Afrikaanse mark. Die lae vlakke van die hefboom onder Suid-Afrikaanse maatskappye is gevind as 'n primêre rede vir die onvermoë van die basiese strukturele modelle om die krediet risiko wat verband hou met die maatskappye te vang. Verder het die uitbreidings van die Merton (1974) model, hoewel prakties 'n uitdaging om te implementeer, kan 'n konsekwente en betroubare wyse waarop die wanbetaling waarskynlikhede bepaal van af die finansiële state en data aandelemark bied.

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CAPM	Capital Asset Pricing Model
CCR	Counterparty credit risk
CDG	Collin-Dufresne and Goldstein
CDS	Credit Default Swap
CVA	Credit valuation adjustment
DD	Distance to default
D&G	Delianedis & Geske
DVA	Debit value adjustment
EDF	Expected default frequency
IFRS	International Financial Reporting Standards
JSE	Johannesburg Stock Exchange
LS	Longstaff & Schwartz
LT	Leland & Toft
MKMV	Moody’s KMV
MTM	Mark-to-market
PFE	Potential future exposure
PD	Probability of Default
S&P	Standard & Poor
OTC	Over the counter

CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

A large number of financial institutions devote considerable resources and efforts towards the measurement and management of credit risk. Credit risk arises from the possibility that borrowers and counterparties in financial transactions may default (Hull, 2012:521). Counterparty credit risk (CCR) also known as default risk, is the risk that a counterparty in an over the counter (OTC) derivatives transaction will default before the expiration date and not be able to meet both current and future obligations arising from the contract. Following the financial crisis of 2008 and the default of major banks and corporations that the idea of risk free and 'too big to fail' can no longer be justified and thus virtually all OTC traded positions bear credit risk in the form of CCR (Gregory, 2012a: 16). Hull (2012:532) mentions that for every derivative position entered into, where the contract is an asset and the counter party thus has an obligation, is exposed to default risk unless it is fully mitigated. The understanding and measurement of the probability of default is therefore essential to the market as a whole because of widespread impact of credit risk.

As of 1 January 2013, the International Financial Reporting Standards (IFRS) 13 requires that fair value of OTC derivatives be measured at fair value that includes the market participants' expectation of the risk of default with regards to both parties involved in the derivatives transaction. As a result, IFRS 13 necessitates entities to incorporate the effects of credit risk when determining a fair value measurement, e.g. by calculating a debit valuation adjustment (DVA) or a credit valuation adjustment (CVA) on their derivatives (Ernst & Young, 2014). The probability of default is critically important in determining CVAs for both financial reporting standards and the valuation of derivatives. Thus in order to calculate CVAs as well as effectively manage, quantify and measure counterparty credit risk various models are required. One of these would be a model that estimates risk-neutral default probabilities, which is a parameter obtained from observed market data.

Estimating the probability of default is essential to the management and handling of credit risk, however making such an estimate is not a simple task. According to Trujillo and Martin (2005) there are in essence three possibilities available to us for estimating/determining the probability of default: using historical experience of default derived from credit ratings; market implied defaults from credit spreads; or lastly, employing some statistical or financial model to derive, from knowledge of a data series, the probability of default. The last alternative is the basis for so-called structural models, which will be the focus of this research paper. The theoretical inspiration for the series of structural models is that of Merton (1974).

The key characteristic of these structural models is that they estimate risk-neutral probabilities of default from financial statement information data over time or use market data of specific variables over time.

1.2 PROBLEM STATEMENT

The changes in financial reporting standards have necessitated that even non-banking entities must account for and incorporate fair value adjustment such as CVA into the valuation of derivative instruments and contracts. The risk-neutral probabilities can be estimated from bond prices and asset swaps or alternatively implied from Credit Default Swap (CDS) quotes (Hull, 2009: 554).

Defining credit spreads from the premiums of single-name Credit Default Swaps (CDSs) instead of bond yields compared to some benchmark would give a more accurate measure of CCR, but CDS data is complex and not readily available (Gregory 2012a: 215). Whether the probability of default is estimated from CDS spreads or bond prices, an estimate of the recovery rate is also required where the recovery rate is the amount the investor is expected to receive if the counterparty in the transaction defaults. Credit spreads are also affected by additional factors such as tax differences, liquidity and recovery rates (Hayne, 2004).

Entities involved in credit and derivative transactions may not have easily accessible bond or CDS data from which to estimate the probability of default. An alternative method to estimate the probability of default is thus required, in order to quantify and manage CCR as well as calculate CVA's in line with financial reporting standards when dealing with counterparties that operate or are publicly traded in such market conditions.

1.3 RESEARCH QUESTION

The research question of the paper is formulated as an attempt to resolve the obstacles/limitations discussed in the problem statement above. The research paper thus aims to answer the question of whether structural models using financial statement information provide consistent and reliable estimates of the probability of default.

1.4 RESEARCH OBJECTIVES

The objective of the research is to explore and critically analyse the various methods and structural models on which the probability of default can be calculated from financial statement information. Using these models the research aims to estimate the probability of default for various companies in the South African market over time. The objective is thus to determine whether these models provide a realistic and consistent estimate for the probability of default.

1.5 RESEARCH BENEFITS

Obtaining a structural model that accurately and consistently estimates risk-neutral default probabilities from financial statement information and market data would mean that there is a relatively simplistic alternative available to modelling default. Such a model would then also naturally provide a manner in which to model CVA and CCR amongst other measures in providing information regarding the credit risk of a particular firm or entity.

Moreover, the structural model uses financial statement and market data that are readily available. This means that the structural models could potentially be used to provide accurate and consistent estimates of the risk-neutral probability of default even where bond and CDS spread market data are not readily available for the counterpart entity being evaluated in the measurement of CCR. This is particularly useful when dealing with counterparties who operate in markets where CDS and bond spread data is not available but rather limited market and financial statement information is available. An example would be developing markets such as Africa.

1.6 RESEARCH DESIGN / CHAPTER OVERVIEW

The research paper has both quantitative and qualitative aspects. The qualitative aspect is the review of the various models and methods for inferring or estimating the default probabilities. The research focus of the paper is more specifically on the structural models that use market and financial statement information in order to estimate or infer the probability of default. Chapter 1 aims to serve as an introduction discussing the background/rationale as well as the context and need for the research.

Chapter 2 of the report contains an outline of the literature that is relevant to the applications and need for estimates of probabilities of default along with the theoretical development of the various methods available for estimating the probability of default. Furthermore, the empirical analysis of previous applications of structural models is also discussed including the South African context.

Chapter 3 then provides the methodology followed in order to obtain the default probabilities of various companies in the South African market over time, for various structural models. The methodology describes how structural models in estimating the probability of default use financial statement information and market data.

Chapter 4 follows by reviewing the results obtained by following the methodology set out in chapter 3. The Merton (1974) and Delianedis and Geske (1998) class of structural models were applied to various companies in the South African market over time in order to compare the results of the probability of default estimated from the various structural models.

These default probabilities are then compared against the Bloomberg probability of default from the proprietary Bloomberg issuer risk model as well as the results of previous studies on the South African market. These comparisons are used in order to provide an indication of whether structural models provide a useful estimate of default probabilities that can be used for derivative valuation, CVA compliance and credit risk management.

A summary of the overall results and outcomes of the research along with the overall conclusion are presented lastly in Chapter 5. This also includes the scope and limitations of the investigation along with recommendations for further research.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, the review of the relevant literature regarding the probability of default and structural models is discussed and analysed in detail. Firstly, a review of the changes in financial reporting standards and regulations that necessitate the need for estimating the probability of default is presented.

Thereafter the ubiquitous methods of the inference of default probabilities from bond prices and CDS spreads market data are discussed and analysed in detail along with inferring default probabilities from historical data. The literature review also importantly distinguishes real-world and risk-neutral probabilities of default. The relationship between corporate bond spreads and credit default swap spreads is also discussed.

Moreover, the literature review explores the theoretical and conceptual basis as well as practical implementation of structural models that infer or estimate default probabilities from market and financial statement information.

2.2 CREDIT VALUATION ADJUSTMENTS

Traditionally, the mark-to-market (MTM) value of a derivative contract was determined by discounting cash flows using the LIBOR curve or country equivalent. However, since the global financial crisis of 2008, accounting and regulatory bodies in the form of IFRS 13 and Basel 3, have recognized the need for true MTM value of a transaction to incorporate the possibility of losses arising from default. Credit Value Adjustment (CVA) / Debit Value Adjustment (DVA) represent the market value of the possibility of loss. The CVA moreover is an objective quantification of Counterparty Credit Risk (CCR) (Bloomberg Treasury and Risk Management, 2015). CVA/DVA is thus the measure used by accounting and regulatory bodies in the attempt to measure credit risk more accurately and provide price transparency in the wake of the 2007 credit crisis.

IFRS 13 requires that CCR be incorporated when determining the fair value of OTC derivatives. However, IFRS 13 does not prescribe an exact valuation method to quantify the adjustments made to the valuation of the derivative position. Thus in practice various methods are applied to quantify CVA where various factors may influence the method an entity chooses to apply. Cost and availability of technology and input data required to model complex credit counterparty risks are but a few of the factors that may influence the method a firm chooses to quantify CVA. (Ernst & Young, 2014).

Two different methods, Potential Future Exposure (PFE) based methods and Current Exposure based methods, have become the prevailing methods of choice for entities over time (Bloomberg Treasury and Risk Management, 2015). PFE based methods use Monte Carlo simulations to create a distribution of future values of the future MTM value of a derivative using market data. CVA/DVA is determined by applying both counterparties default probabilities to the distribution of future values of the derivative. PFE based methods are widely regarded as the most quantitatively complex and accurate method.

Alternatively, Current Exposure based methods determine CVA/DVA based only on current market information available such as interest rate or forward curves. Audit firms have orated that Current Exposure based methods, although practically easier to implement, in certain circumstances do no accurately reflect the market value of CCR (Numerix, 2012).

CVA significantly changes the manner in which derivative valuation is viewed as CVA essentially invalidates certain underlying assumptions behind risk-neutral valuation in terms of market completeness and availability of a hedge to create a riskless portfolio (Numerix, 2012). Given certain simplifying assumptions, Gregory (2012a) shows that where risk free valuation is possible, and the probability of default and exposures are independent, it is possible to derive the following equation for CVA:

$$CVA \approx (1 - Rec) \sum_{i=1}^{m} DF(t_i) EE(t_i) PD(t_{i-1}, t_i) \tag{2.1}$$

Where $(1 - Rec)$ is the loss given default, DF is the relevant discount factor, EE represents the expected exposure in the period and PD is the marginal probability of default in the relevant time horizon (Gregory, 2012a:243). Furthermore, IFRS 13 requires that valuation techniques maximise the use of observable inputs and minimize the use of unobservable inputs, thus IFRS 13 would generally require the use of observable market credit spreads where available (Ernst & Young, 2014). Thus, current market observable credit spreads are preferred to historical or blended data as sources of credit risk data for measuring CVA.

The probability of default is a major determinant in the accurate quantification of CVA/DVA as well as effectively measuring and managing counterparty credit risk. IFRS 13 reiterates the evident need for a parsimonious procedure to determine accurately the probability of default, and hence quantification of CVA and improved credit risk management.

2.3 HISTORICAL DEFAULT PROBABILITIES

Real-world or physical default probabilities are determined from analysis of historical default data usually published by rating agencies. Vassalou and Xing (2004) note that the biggest disadvantages of the historical approach is that historical accounting data provides no information regarding future expectations of the firm or volatilities of the firm’s assets.

2.3.1 Credit ratings

Credit rating agencies are in the business of providing an opinion or measure around the credit worthiness of a particular financial instrument or issuer. Rating agencies thus provide a measure of the likelihood that a particular issuer or instrument will not be able to meet the financial obligations of the contract. The three largest credit rating agencies are Fitch, Moody’s and Standard & Poor’s (S&P) (Langohr and Langohr, 2008).

Moody’s (2014) defines the credit rating as “an assessment of the ability and willingness of an issuer of fixed-income securities to make full and timely payment of amounts due on the security over its life.”

Table 2.1: Moody’s Global scale ratings descriptions

Aaa	Obligations rated Aaa are judged to be of the highest quality, subject to the lowest level of credit risk.
Aa	Obligations rated Aa are judged to be of high quality and are subject to very low credit risk.
A	Obligations rated A are judged to be upper-medium grade and are subject to low credit risk.
Baa	Obligations rated Baa are judged to be medium-grade and subject to moderate credit risk and as such may possess certain speculative characteristics.
Ba	Obligations rated Ba are judged to be speculative and are subject to substantial credit risk.
B	Obligations rated B are considered speculative and are subject to high credit risk.
Caa	Obligations rated Caa are judged to be speculative of poor standing and are subject to very high credit risk.
Ca	Obligations rated Ca are highly speculative and are likely in, or very near, default, with some prospect of recovery of principal and interest.
C	Obligations rated C are the lowest rated and are typically in default, with little prospect for recovery of principal or interest.

Source: Moody’s Investors Service 2014:5

Table 2.1 summarizes the various credit ratings and meanings issued by Moody’s.

The credit grade of the particular instrument (usually bonds) or issuer is even further classified into investment grade, which is Baa and above for Moody's, and speculative grade, which compromise the class of rating scores Ba and below (Langohr and Langohr, 2008). Moody's also have national long term ratings designed for different countries which provide an indication of the creditworthiness of the issuer or obligation relative to other domestic issuers within the country (Moody's, 2014). The last two letters in national ratings indicate the country of the issuer.

Rating agencies typically then use average cumulative default rates as per rating class in order to determine historical default probabilities within each rating class. For investment grade bonds the probability of default in a given year is an increasing function, this is because the bond issuer is initially considered creditworthy, however as more time elapses, the greater the chance of deterioration in the creditworthiness of the issuer. The converse is true for speculative grade bonds, as the longer the issuer survives, it becomes more likely that the issuer will show improved financial prospects and have overcome the initial questionable creditworthiness. (Fons, 1994).

2.3.2 Hazard rates

The default probabilities referred to above are known as the unconditional default probability. It is the probability of defaulting during any given year as seen at time zero. The probability of default during any given year conditional on no default in any prior period is referred to as the conditional default probability or hazard rate, also known as default intensities (Malz, 2011).

The hazard rate $\lambda(t)$ at time t is defined such that $\lambda(t)\Delta t$ is the probability of default between time t and $t + \Delta t$ under condition that no default has occurred prior to time t. If $V(t)$ is the cumulative probability of the firm surviving until time t and $Q(t)$ is the probability of default by time t, Hull (2012: 523) derives the following:

$$V(t) = e^{\int_0^t \lambda(\tau) d\tau} \tag{2.2}$$

$$Q(t) = 1 - e^{\bar{\lambda}(t)} \tag{2.3}$$

Where $\bar{\lambda}(t)$ is the average hazard rate or default intensity in the interval [0,t].

2.3.3 Risk-neutral default probabilities vs. real-world default probabilities

There is an important distinction to be made between real-world or historical default probabilities and risk-neutral default probabilities. Hull (2012: 528-529) defines risk-neutral probabilities of default as those determined where expected losses can be discounted at the risk-free rate in a risk-neutral world as per the risk-neutral valuation principle. Contrastingly, real-world default probabilities are those implied from historical data and as such sometimes referred to as 'physical probabilities'.

A commonly noted feature of credit markets is the large discrepancy between default probabilities calculated from historical data and default probabilities implied from bond prices or credit default swaps (Hull, Predescu and White, 2004). Altman (1989) was one of the first researchers to note this discrepancy and showed that, even after considering the impact of default, an investor could earn significantly more than the risk-free rate on average by holding corporate bonds.

Hull (2012: 528-529) suggests that bond traders do not base their price solely on the actual probability of default and build in an extra return to compensate for additional risks they are bearing in the trading position. The excess returns observed on corporate bonds are also affected by additional factors such as tax differences, liquidity and recovery rates (Hayne, 2004). Hull (2012: 528-529) along with Amato and Remolona (2003) proposes that the most important reason for the difference in return, is the non-systematic risk associated with each bond as bond defaults do not occur independently and non-systematic risk is difficult to 'diversify away' from. This can be seen as default probabilities move very well together in different periods according to macro-economic conditions. Thus, bond traders may earn additional return for bearing both systematic and non-systematic risk, contributing to the hefty difference in actual default probabilities and risk-neutral default probabilities.

The use of risk-neutral or actual default probabilities in the credit analysis depends on the purpose of the analysis. Real-world default probabilities should be used in profit and loss scenario analysis and determining banking capital requirements (Hull *et al.*, 2004:1). Gregory (2012a: 198) describes risk-neutral probabilities of default as estimates of the market price of default rather than estimates of actual default probabilities.

IFRS 13 provides that fair value is a market-based measurement requiring that risk-neutral default probabilities be used in the fair value measurement of an OTC derivative (Ernst & Young, 2014). Furthermore, this implies that risk-neutral default probabilities should be used when valuing credit derivatives and estimating the potential impact of default on the pricing of instruments (Credit Value Adjustment/ Debit Value Adjustment).

2.4 DEFAULT PROBABILITIES FROM CREDIT SPREADS

Corporate bonds on average trade at higher yields than similar risk free government or treasury bonds. This yield spread is partly due to the credit risk of corporate bonds and thus often referred to as the credit spread (Huang and Huang, 2003).

2.4.1 Default probabilities from bond prices

Under the assumption that the yield spread on the corporate bond is only owing to the compensation for the possibility of default, Hull *et al.* (2004) shows that the hazard rate or default intensity can be estimated from bond prices as follows:

$$\bar{\lambda} = \frac{s}{1-R} \tag{2.4}$$

Where s is the yield spread of the corporate bond over similar risk free bond and R is the expected recovery rate. The assumption is far from realistic as in practice many other factors contribute to the credit spread such as liquidity, embedded options and tax treatments of the instrument (Huang and Huang, 2003).

A key determinant of default probabilities from bond prices is the meaning of the risk-free rate or risk free bond against which the credit or yield spread is determined. Duffee (1996) noted that the treasury rate is lower than similar very low credit risk rates for a variety of factors and that the treasury rate no longer provided a suitable proxy for the risk-free rate. The tendency of treasury rates to be lower than other rates has led many market participants to regard the swap rate as an improved proxy for the risk-free rate (Hull *et al.*, 2004: 3).

The Credit Default Swap (CDS) market provides a manner in which the benchmark risk-free rate used by participants in credit markets can be estimated. CDS are considered less influenced by non-default factors and thus able to provide a good proxy of the risk-free rate when analysing default risk (Wang, 2006).

The other key variable in determining default probabilities from bond prices as per equation 2.4 is the expected recovery rate. The expected recovery rate for a bond is usually expressed as the bond's market value shortly after defaulting, as a percentage of its face value (Hull, 2012: 523). The expected recovery rate is thus the percentage of the original investment that an investor expects to receive in the event of default.

There are varieties of factors that influence the expected recovery rate for a bond however, Fons (1994) argues that the chief determinant of the expected recovery rate is the bond's seniority within the capital structure of the firm. Moody's estimates the recovery rates of bonds by seniority, based on bond prices one month after default. The estimation of default probabilities from bond prices and yield spreads thus requires some form of a subjective or historical estimate for the expected recovery rate.

In most studies surrounding the extracting of default probabilities from bond prices and credit spreads, such as the works of Jarrow and Turnbull (1995) along with Duffie and Singleton (1999), only plain vanilla bonds are considered in the study as inferring default probabilities from bonds with embedded options or floating rates become significantly more complex.

Estimating default probabilities for a firm from the bonds it has issued, becomes problematic for firms that issue a variety of types of bonds in addition to the plain vanilla type bonds.

Another difficulty encountered by this approach is the inability to separate, easily, the portion of the credit spread owing to default and the part owing to the rate of recovery. Furthermore, the findings of Elton *et al.* (2001) along with Delianedis and Geske (2001) indicate that default risk only accounts for a small proportion of the yield spread and that the greater part of the credit spread can be attributed to fiscal and systematic risk effects. This is consistent with the reasoning for the significant difference between actual default probabilities and risk-neutral default probabilities described in the previous section.

2.4.2 Credit Default Swap Spreads

A CDS is a popular credit derivative that provides insurance against the risk of default by a particular firm or for a specified corporate bond known as the reference entity and reference obligation respectively. The buyer of the insurance has the right to sell the bonds of the company at face value to the insurance seller, should a credit event occur. The total value of the bonds that can be sold is referred to as the notional principle of the CDS (Wang, 2006).

A key aspect of a CDS contract is the definition of the credit event that triggers the CDS. Most typically, a credit event defined in a CDS contract includes bankruptcy, failure to make payment and any form of restructuring of debt obligations by the reference entity that is adverse to creditors (Liang *et al*., 2010). Typically, the buyer of the CDS makes periodic payments to the seller until the end of the life of the contract or a credit event occurs.

A vanilla or plain CDS contract usually specifies two potential cash flow streams in the form of a fixed premium leg and contingent leg (Wang, 2006). The value to the CDS contract to the buyer of default insurance is thus the difference in expected present value of these two potential cash flow streams. Liang *et al*. (2010) provides the following equations for valuing a CDS contract:

$$Value\ of\ CDS = E[\ PV\ (contingent\ leg)] - E[\ PV\ (Fixed\ premium\ leg)\ ] \qquad (2.5)$$

Where:

$$E[\ PV\ (Fixed\ premium\ leg)\ ] = \sum D(t_i)q(t_i)Sd + \sum D(t_i)\{q(t_{i-1}) - q(t_i)\}S\times {}^{d_i}/_2$$

$$E[\ PV\ (contingent\ leg)\ ] = (1-R)\sum D(t_i)\{q(t_{i-1}) - q(t_i)\}$$

Where $D(t)$ is the relevant discount rate at time t, $q(t)$ is the survival probability at time t, S represents the annual premium and d is the accrual days.

The CDS spread is defined as the total of payments made in a year as a percentage of the notional principal. The spread, S, is set initially so that the value of the CDS is zero at origination of the contract (Liang *et al*., 2010).

$$S = \frac{(1-R)\sum D(t_i)\{q(t_{i-1}) - q(t_i)\}}{\sum D(t_i)q(t_i)d_i + D(t_i)\{q(t_{i-1}) - q(t_i)\}\,{}^{d_i}/_2} \tag{2.6}$$

Thus in order to value CDSs, estimates of risk-neutral default probabilities and the recovery rate are required for the reference obligation. Alternatively, Duffie (1998) shows that the asset-swap spread and the term structure of risk-free rates can be used together in order to estimate the CDS spread. Under certain assumptions the CDS spread and the par asset-swap spread are exactly the same for small default probabilities, however it is dangerous to assume that the asset-swap spread is a reasonable proxy for the CDS spread in the case of premium discount bounds (Duffie, 1998).

The CDS can be used to hedge a position in a corporate bond and following no arbitrage arguments, the n-year corporate bond yield spread or credit spread should be approximately equivalent to the n-year CDS spread otherwise an arbitrage opportunity exists (Hull, 2012: 550). The work of Hull (2012), Duffie (1998) and Wang (2006) indicates that bond market spreads should give a roughly similar spread to those obtained from the CDS market.

Given the pricing methodology for CDS contracts as outlined above, it can be seen that risk-neutral default probabilities can be implied from market CDS quotes. This is achieved in a similar manner to how implied volatility is determined from prices of actively traded options (Hull, 2012: 554). Additionally, the CDS spread provides a good indication of the creditworthiness of the counterparty from the market participant’s view. However, CDS market data is not readily accessible or available for most smaller or private entities and CDS quotes may include liquidity premiums due to low trading volumes. (Ernst & Young, 2014).

2.5 STRUCTURAL MODELS OF DEFAULT PROBABILITY

Many early approaches to modelling credit risk took a statistical route in order to distinguish defaulters from non-defaulters such as the discriminant analysis approach of the Altman-Zeta model (Trujillo and Martin, 2005). In 1974, Robert Merton introduced a new option-theoretic approach to credit risk modelling and measurement based on ideas and formulations that were implicit in the Black and Scholes (1973) option-theoretic framework.

The class of models that has developed around the Merton (1974) approach is presently known as the class of ‘structural models’. The basis of the structural approach is that the debt and equity of a firm can be regarded as contingent claims on the firm’s assets.

The value of the debt and equity of a firm thus depends on the value of its assets as well as the forward-looking expectation surrounding the value of those assets.

2.5.1 Merton (1974) model

The firm value in the context of the Merton (1974) model is the economic value of the total assets of the firm. As with all structural models, the Merton model begins with a specification of a stochastic process for the firm value. The Merton model assumes that the firm value follows a geometric Brownian motion:

$$\mathrm{d}V_t = \mu V_t \mathrm{d}t + \sigma V_t \mathrm{d}W_t \tag{2.7}$$

Where V_t is the firm value at time t, μ is the drift of the firm value, and σ is the volatility of the firm value. The second assumption of the Merton model is that the capital structure of the firm consists solely of equity and debt. Furthermore, the debt is assumed to be a single issue of zero-coupon form where the face value of the debt is denoted by D and the maturity date is T.

To complete the model, further assumptions regarding the conditions that trigger default and the costs incurred in the event of default are required. The Merton (1974) model assumes that default can only occur at time T when the debt becomes due and no covenants can trigger default before time T. Furthermore, debt holders are assumed to have absolute priority over equity holders in the event of default and there are no frictional market costs associated with liquidation in the event of default.

Under these assumptions Merton (1974) shows that holding the risky debt of the firm is equivalent to holding a portfolio consisting of a long position in default risk free bond paying D at time T and short a put on the firm's assets with strike D and maturity T. The following decomposition follows naturally:

$$B^* = B - P \tag{2.8}$$

Where B^* represents the value of risky debt, B is the value of riskless debt and P is the value of the put on the firm's assets. This decomposition importantly shows that the spread on the risky debt is completely determined by the value of the put, P (Smit, Swart and Van Niekerk, 2003). The value of the put can be determined using the Black-Scholes formula since all the conditions of Black-Scholes have been met in the assumptions. Merton (1974) expresses the value of the put slightly differently to the standard Black-Scholes formula:

$$P = e^{-r(T-t)}\, D.N\left(-d + \sigma\sqrt{T-t}\right) - V_t N(-d) \tag{2.9}$$

Where:

$$d = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left({}^{1}/_{L}\right) + \frac{1}{2}\sigma^2\,(T-t)\right]$$

$$L = \frac{e^{-r(T-t)}\,D}{V_t}$$

In addition, $N(.)$ is the standard normal distribution function. The risk-neutral probability of default is easily extracted as the probability that $V_t < D$. From Black-Scholes formula, this is simply the probability that the put P finishes 'in the money'. Smit *et al.* (2003) show that the risk-neutral probability of default is given by:

$$N(-d + \sigma\sqrt{T-t}) \tag{2.10}$$

The actual or real-world probability is given similarly as the probability that $V_t < D$. However, the process for the firm value has drift μ as opposed to drift in the risk-neutral world where the risk-free rate, r, is the drift of the firm value process. The actual probability of default will typically be less than risk-neutral probabilities since $\mu > r$ usually. The higher risk-neutral default probability can be interpreted as comprising of actual default probability and a premium for uncertainty of timing and magnitude of the default (Sundaran and Das, 2010).

Another useful feature of the Merton (1974) model is that is allows for estimation of expected recovery rates in the risk-neutral setting. Under the Merton framework, Smit *et al.* (2003) provide the following closed form expression for the expected recovery rate:

$$\frac{1}{D}E_T[V_T|\,V_T < D] = e^{-r(T-t)}\left(\frac{V_T}{D}\right)\left(\frac{N(-d)}{N\left(-d + \sigma\sqrt{T-t}\right)}\right) \tag{2.11}$$

This feature is extremely useful since both CDS spreads and bonds prices require an estimate of the recovery rate to estimate default probabilities. Although the model is theoretically very appealing, since it provides a simplistic model for the credit spread along with default probability and recovery rates, the model encounters a number of major challenges in practical implementation. The first of these challenges is that both the firm value V_t and its volatility σ are unobservable in the market. Wang and Suo (2006) argue that in the Merton model, the firm's equity is treated as a European call option on the firm's assets and hence the firm value and volatility should satisfy the following set of simultaneous equations:

$$E_T[V_T,\sigma] = V_T\,N(d) - e^{-r(T-t)}\,D\,N(d - \sigma\sqrt{T-t}) \tag{2.12}$$

&

$$\sigma_E = \sigma V_T\frac{N(d)}{E_T} \tag{2.13}$$

The relationship between the equity and asset volatility only holds instantaneously and the algorithm forces stochastic variables to be constant, where in practice the hedge ratio and leverage ratio are not stable enough to provide meaningful estimates (Holman *et.al,* 2011). Crosbie and Bohn (2003) illustrate that the procedure biases the probability of default in exactly the wrong direction as increased leveraging will drive down asset volatility and under predict default.

Vassalou and Xing (2004) describe a more complex iterative procedure to solve for the asset volatility. Alternatively, Duan (1994) describes an intricate maximum likelihood approach based on observed market equity or bond prices in order to solve the unknown parameters relating to the firms value and volatility. A distinct advantage of the maximum likelihood approach is that it directly provides an estimate of the real-world drift parameter, μ, of the unobserved asset value process under the physical probability measure (Wang and Suo, 2006).

The second major issue encountered with implementing the Merton (1974) model is that the capital structure assumption is too simplistic. In practice, capital structures consist of many issues of debt outstanding, with varied coupons, maturities and subordination structures (Sundaran and Das, 2010). In order to simplify reality, Delianedis and Geske (1998) suggest a zero-coupon bond that has an equivalent duration of the existing structure replacing the capital structure. An alternative in the popular Moody's KMV vendor model is to use the aggregate of short term and long-term liabilities to estimate the face value of the zero-coupon debt D.

2.5.2 Delianedis and Geske (1998) model

As opposed to simplifying capital structures to fit within the existing Merton model framework, Sundaran and Das (2010) suggest that extending the theoretical structure of the model to incorporate more complex debt structures is the more economically correct manner in which to solve the problem of capturing the effects of more complex capital structures.

Delianedis and Geske (1998) (D&G) provide extensions of the Merton model that allows for more complex capital structures. These models allow for multiple debt issues of varying coupons, maturities and seniority or subordination (Chen, 2013).

In the most simple extension of the Merton Model, D&G (1998) allow for two tranches of zero-coupon debt in the firm's debt structure with face values D_1 and D_2 and maturities T_1 and T_2 respectively where $T_1 < T_2$. Since there are now two dates at which equity holders may choose to default the D&G model thus involves a compound option pricing approach (Sundaran and Das, 2010).

Delianedis and Geske (1998) further illustrate that at the first maturity date T_1, the firm is solvent if:

$$V_{T_1} > D_1 + B_{2,T_1} \tag{2.14}$$

Where V_{T_1} is the value of the firm's assets at T_1, D_1 is the face value of the first tranche of debt at T_1 and B_{2,T_1} is the value of the second tranche at T_1. If the firm is solvent, the Delianedis and Geske (1998) model then assumes that the first tranche of debt will be refinanced with equity. The model may be implemented under the assumption that refinancing is not allowed however, this adversely affects the second tranche of debt and is less realistic (Sundaran and Das, 2010).

The condition for solvency provided by Delianedis and Geske (1998) defines a critical cut-off value V^*, for the value of the firm at T_1, which is equivalent to the strike price of the first option in a compound option (Chen, 2013). The critical cut-off value or strike price of the first option is given by:

$$V^* = D_1 + B_{2,T_1} \tag{2.15}$$

Whilst the strike price for the second option at date T_2 is simply the face value of the second tranche of debt, D_2 (Sundaran and Das, 2010). Delianedis and Geske (1998) shows that using the strike prices of the two options as described above, the price of equity today can be treated as the value of the compound option with such exercise prices and provides the following solution:

$$E_T = V_T N_2\left[d_1 + \sigma\sqrt{T-t}; d_2 + \sigma\sqrt{T-t}; \rho\right] - D_2 e^{-r(T_2-t)}\, N_2[d_1; d_2; \rho] - D_1 e^{-r(T_1-t)}\, N(d_1) \tag{2.16}$$

Where:

$$\rho = \sqrt{\frac{T_1 - t}{T_2 - t}}$$

$$d_1 = \frac{\ln\left(\frac{V_t}{V^*}\right) + \left(r + {}^1\!/_2\, \sigma^2\right)(T_1 - t)}{\sigma\sqrt{T_1 - t}}$$

$$d_2 = \frac{\ln\left(\frac{V_t}{D_2}\right) + \left(r + {}^1\!/_2\, \sigma^2\right)(T_2 - t)}{\sigma\sqrt{T_2 - t}}$$

And $N_2[.]$ is the cumulative bivariate standard normal distribution with correlation coefficient ρ.

From the compound option model, Delianedis and Geske (1998) provide three risk-neutral probabilities as follows:

$$\text{risk neutral short run PD} = 1 - N(d_1) \tag{2.17}$$

$$\text{risk neutral long run PD} = 1 - \frac{N_2[d_1; d_2, \rho]}{N(d_1)} \tag{2.18}$$

$$\text{risk neutral total PD} = 1 - N_2[d_1; d_2; \rho] \tag{2.19}$$

The short run default probability represents the probability of default at T_1. The total default probability represents the probability of the firm defaulting at either T_1 or T_2. The long-term default probability is the probability of default at T_2 conditional on not having defaulted at T_1 and is thus also refered to as the forward default probability (Chen, 2013).

The D&G model has the appealing feature of being able to capture both short-term and long-term default characteristics of the firm simultaneously. Sundaran and Das (2010) argue that there are many firms with poor quality yet, conditional on survival of initial financial difficulty, have reasonable longer term financial prospects and that the forward default probability of the D&G model is likely to reflect these key features.

Although the model appears to be a relatively simply extension of the Merton framework, considerable additional complexity arises in solving for the unobservable parameters of process for value of the firm. Delianedis and Geske (1998) shows that the unobservable parameters $\{V_t, \sigma, V^*\}$ can be estimated from the system of equations 2.15, 2.16 and 2.20 below.

$$\sigma_E = \sigma V_T \frac{N(d_1)}{E_T} \tag{2.20}$$

The procedure for estimating these unobservable parameters is subject to the same weaknesses as with the case of Merton. The equity and asset volatility relationship is still instantaneous as described by Crosbie and Bohn (2003), additionally there is the added complexity of a third unobservable variable V^*, the cut-off value, in the estimation procedure.

2.5.3 Practitioner models

One on the most notable implementation of a structural credit risk measurement model is the Moody's KMV (MKMV), Trujillo & Martin (2005) summarizes the MKMV approach in four stages:

- (i) Calculate a default boundary.
- (ii) Estimate asset value and volatility.
- (iii) Calculate the Distance to Default (DD).
- (iv) Map DD into Expected Default Frequency (EDF).

The first two stages of the MKMV approach are analogous to that of the Merton approach. In the first stage, the capital structure is collapsed into a single debt issue or default boundary calculated as the sum of the short-term liabilities and a fraction of the longer-term liabilities. In the second stage, the asset value and volatility are backed out from observed equity value, volatility and capital structures. This is achieved using a proprietary variant of the Black and Scholes/ Merton option-pricing model (Crosbie and Bohn, 2003).

In the third stage the MKMV approach moves away from the Merton approach and defines the 'distance to default' as the number of standard deviations the firm value has to move make before the firm is in default (Hayne, 2004). The MKMV approach defines the distance to default δ in a simplified manner as shown by Crosbie and Bohn (2003):

$$\delta = \frac{V_t - D}{\sigma V_t} \tag{2.21}$$

The ratio δ represents the number of standard deviations the firm is from default. Sundaran and Das (2010) illustrate that normalizing the distance in this fashion allows for comparability between firms of how far the firm is from default even though the firms may differ substantially in other ways. The final stage uses the estimated 'DD' to determine an 'expected default frequency' (EDF), from a proprietary default database, which represents the likelihood of the given firm defaulting over the specified horizon (Hayne, 2004). The MKMV practioner model thus uses a blend of market and historical data in a structural framework to estimate the probability of default for a given firm.

Another priopertery model of great use is the Bloomberg issuer risk model. According to Bloomberg (2012) the issuer risk model provides an independent assesment of credit health, using market and fundamental data with innovative quantitative models. The bloomberg issuer risk model provides one and five year default probabilities along with implied CDS spreads. In this paper, the Bloomberg issuer risk model is assumed to provide reasonable and consistent estimates for default probabilities and can thus be used as stable benchmark for comparison of default probabilities estimated by structural models.

2.5.4 Extensions of the Merton model

The original Merton model proposes a number of simplifying assumptions in order to apply the option-pricing framework, these assumptions however present an over simplified picture of reality and impose a variety of potential restrictions on the model (Sundaran and Das, 2010). Over time, numerous extensions of the original Merton framework have been developed in an attempt to address the various limitations of the original Merton Model.

Trujillo and Martin (2005) cite the most important restriction of the Merton model as the assumption that the firm has only a single issue of outstanding debt and that insolvency can only occur when such obligations become due. Black and Cox (1976) provide one of the earliest extensions of the Merton framework by allowing for default before maturity if the value of the firm falls below an endogenous default barrier. The endogenous default barrier is determined as the value of the firm under which the managers of the firm are incentivised to default in order to maximize the value of the equity (Hayne, 2004). These approaches are known as first-passage models.

The Merton framework assumes that the risk-free rates of interest are deterministic which is a clearly unrealistic and limiting assumption since unanticipated changes in the interest rate can significantly impact the value of debt (Trujillo and Martin, 2005). Shimko *et al.* (1993) present a generalization of the Merton model to include for stochastic interest rates that follow from the model of Vasicek (1977). The Black and Cox (1976) model is also extended to include stochastic interest rates in the works of Longstaff and Schwartz (1995) (LS). Leland and Toft (1996) (LT) further extend the approach of Longstaff and Schwartz (1995) by deriving the optimal endogenous default value for firms issuing debt of arbitrary maturities and introducing tax benefits of debt into the model.

The Merton model assumes away any dead weight cost of bankruptcy and no potential renegotiation of debt is allowed contrary to what is commonly observed. Huang and Huang (2003) noted that many firms continue to operate with negative net worth and that implied default costs must be extremely high to explain relatively low recovery rates on corporate bonds. Anderson and Sunderason (1996) extend the Merton and D&G frameworks to allow for costs of bankruptcy and the possibility of renegotiation of debt. Anderson and Sunderason (1996) argue that the presence of these costs and the possibility of renegotiation provides equity holders with incentive to participate in strategic debt service and continue to operate with negative net worth in certain instances.

Alexander (2008) shows that emperical evidence suggests that equity returns particularly tend to be distributed leptokurtically and heavily skewed with fat tails. Zhou (1997) adresses the issue of large tails in returns by modelling the firm value process as a jump-diffusion.

Zhou (1997) shows that the jump-diffusion process also allows for the recovery rates estimated in the model to be naturally stochastic. Furthermore, notable extensions of the Merton framework include Collin-Dufresne and Goldstein (2001) (CDG) who note that firms may have specific target captial structures and incorporate mean-reverting leverage ratios and stochastic interest rates into the model.

2.6 EMPERICAL PERFORMANCE OF STRUCTURAL MODELS

The structural model approach makes use of a sound economic basis and more importantly makes use of current market prices in its implementation, more specifically information from equity markets which tend to be more liquid and informative than credit markets (Sundaran and

Das, 2010). It has been comprehensively shown that the structural model approach not only provides a model for default but includes many other useful outputs such as naturally defined risk-neutral recovery rates and credit spreads of corporate debt to name a few.

Arora, Bohn and Zhou (2005) note that although the structural approach provides an appealing conceptual and theoretical framework, the practical applicability of many of the generalizations and extensions of the Merton model is limited. While emperical evidence is still scarce, more and more emperical researchers have started testing these model extensions and generalizations.

2.6.1 Credit spreads

There have been numerous studies to discern whether structural models are able to explain observed corporate yield spreads. Most notable among these are the works of Huang and Huang (2003) and Eom *et al.* (2004) who provide a comprehensive comparison among several structural models.

Huang and Huang (2003) find that the yield spread implied by structural models are substantially underestimated in comparison to observed yield spreads, especially for short maturities and investment grade bonds. Eom *et al.* (2004) finds that the Merton (1974) and D&G (1998) models significantly under predict the yield spread. Furthermore, Eom *et al.* (2004) shows that the extensions of the Merton framework tend to over predict spreads for firms with high leverage or volatility with the exception of the LT (1996) model and the structural models appear to overstate the credit risk of risky bonds while simultaneously underestimating the risk of safer bonds.

Huang and Huang (2003) explain this mixed performance of the structural models by arguing that while default risk can account for a large portion of the credit spread for low-grade debt, the default risk only accounts for a small portion of the spread in investment grade debt. The remaining portion of the spread is attributed to systematic risk effects (Vassalou & Xing, 2004).

2.6.2 Default probabilities and credit ratings

The structural model approach is rarely used in the valuation of different tranches of corporate debt and does not facilitate the valuation of most credit derivatives but rather is primarily used as an indicator or predictor of distress (Sundaran and Das, 2010). There is substantial evidence indicating that structural models perform well in default prediction.

Wang and Suo (2006) show that when using equity prices as the input to the maximum likelihood estimation procedure of Duan (1994), one-year default probabilities from the original Merton (1974) model are close to zero for most of the investment grade firms. Alternatively, Wang and Suo (2006) find that when using bond prices the estimation process does not converge for most firms in the sample.

The low default probabilities estimated are consistent with Huang and Huang (2003) since the small credit spread for investment grade firms also implies low default probabilities. Hull (2012: 531) suggests that the Merton model and its extensions provide good rankings of default probabilities which can be transformed into useful real-world or risk-neutral default probabilities using a monotonic transformation. Delianedis and Geske (1998) further find that, empirically, the compound option approach was able to forecast rating transitions accurately.

Moreover, Wang and Suo (2006) find that the performance of the Merton model is significantly improved when assuming a stochastic interest rate structure. Wang and Suo (2006) find that in contrast to the structural models inability to explain credit spreads, the default probabilities from the LS and LT models are very close to real world observations. However, Hayne (2004) finds that shorter-term default frequencies for these models tend to be underestimated suggesting that a jump component should be included in the asset value process. The CDG model with a mean-reverting capital tends to over-predict default probabilities largely and is highly sensitive to the choice of interest rate parameters (Huang and Huang, 2003).

Tudela and Young (2003) combine the Merton model with additional financial information to form a hybrid model and find that such an implementation is able to provide a strong signal of failure or default over a one-year time horizon. Tudela and Young (2003) find default probabilities had a mean of 47.3% for firms that went bankrupt versus 5.4% for those who did not default. This importantly shows that the structural model approach is successfully able to distinguish between defaulting firms from non-defaulters. Using a sample of observed defaults, Sundaran and Das (2010) further show that the distance to default metric produced by the structural model approach can be used to create a cumulative accuracy profile (CAP) that measures the forecast validity produced by the distance to default measure within the sample of firms. Depending on the specifics of the model and the universe of firms, the structural model approach produces accuracy ratios varying from $65-90\%$ in default prediction.

Wang (2006) suggests that the Merton model and its extensions are able to capture the characteristics of the firms' credit risk adequately, however fail to price corporate bonds owing to the additional factors influencing the credit spread.

2.6.3 South African context

According to Holman *et al*. (2011), the South African market provides a rather unique case with respect to default estimation as no firm has defaulted on their listed debt. This would imply a historic default probability of zero for any firm, which theoretically cannot be true.

Since the classical historical approach to default estimation is not applicable, the works of Smit *et al*. (2003), Venter and Styger (2008) and Holman *et al*. (2011) are amongst the most notable studies to determine whether the structural approach provides a viable alternative to estimating default probabilities in the South African market.

In a study of a range of 20 South African companies, rated AAA to BBB, Smit *et al*. (2003) found similarly to research conducted in the rest of the world that the Merton model produced spreads and hence default probabilities that were too small. The study of Smit *et al*. (2003) showed clearly that the credit spreads and hence default probabilities increased with asset volatility and leverage ratios. More notably the study found that when applying the model of Shimko *et al.* (1993) by allowing for stochastic interest rates, the results produce credit spreads that match observed spreads much better and the model is more responsive to low volatilities and high leverage ratios.

Holman *et al*. (2011) estimated default for the top 42 firms on the Johannesburg Stock Exchange (JSE), excluding financial firms, using the Merton (1974) framework. The study interestingly finds that the distance to default as produced by the Merton model is so large that it suggests South African firms may have sub-optimal capital structures otherwise the Merton model is not adequately reflecting default risk. Holman *et al*. (2011) suggests that the low levels of leverage amongst South African firms may be due to limited growth opportunities and hence firms do not need to issue debt in order to finance expansion. Furthermore, it is this low level of leverage amongst South African firms that is responsible for significant under prediction of default probabilities. This is evidenced in the study of Holman *et al*. (2011) as the only two firms that produced default probabilities significantly different to zero, were significantly more leveraged than the remaining firms in the sample.

Holman *et al*. (2011) further evaluates whether there is a relationship between the credit ratings of the firms and the default probabilities as calculated by the Merton model. Contrary to Hull (2012), the study found no apparent relationship between the Merton model default rankings and the credit ratings of the firms as issued by Moody's or Fitch, with some firms having better credit ratings exhibiting larger default probabilities than poorly credit rated firms.

In a unique application of the structural approach, Venter and Styger (2008) modify the Merton model in order to make it more readily applicable to banking firms within South Africa. Venter and Styger (2008) shows that the assumptions of the Merton (1974) regarding the treatment of the firms' liabilities make it unsuitable for a direct application to the banking sector in South Africa.

In essence, the modification made by Venter and Styger (2008) assumes that both assets and liabilities of the firm follow a geometric Brownian motion and are correlated. Under this modified assumption the call option view on the equity of the firm may be replaced by suitable swap option views. Venter and Styger (2008) fit this model to three leading banks in South Africa and find that the implied buffer capital levels and failure probabilities produced from the models provide useful and reasonable comparative risk measures.

The empirical studies suggest that the Merton (1974) model should be used as a limited source of information regarding credit risk in South Africa. However, it has been shown that the extensions of the Merton model can be used to provide more useful and realistic information regarding the credit risk of firms in South Africa.

2.7 SUMMARY

It has been comprehensively shown that the probability of default plays an essential role in multiple facets of the credit risk management process. The default probability is clearly not only a major determinant in default prediction and measurement of CCR but is also critical to performing required regulatory valuation adjustments to derivatives in the form of CVA.

Estimating default probabilities can be separated into three distinct approaches in the forms of the historical approach using historical default data or credit ratings, estimating default probabilities from observed bond and CDS spreads and lastly, the structural model approach that uses market and financial statement data.

IFRS 13 dictates that default probabilities from observed bond or CDS spreads should be used for CVA quantification when available as the CVA is intended to be a market based measure. Bond and CDS market data are however not always readily available since the number of firms whose debt is traded in organised markets is substantially lower than the number of firms whose security prices are quoted in such markets. The structural approach thus provides a valuable alternative to quantifying CVA where bond and CDS market data is not available as the models produce estimates of default using widely available security market data and publicly available financial statement information. Furthermore, the use of observed equity prices in the structural approach is consistent with IFRS that CVA is a market-based measure and the approach is thus preferred to the historical approach since the historical approach does not include market expectations surrounding the future progress of the firm.

It has been seen that the structural approach has difficulty in explaining the observed credit spreads on bonds for numerous reasons. Furthermore, the Merton model appears to offer very limited insight with regards to the credit risk of firms in the global and South African context.

The extensions and generalizations of the Merton model, although considerably more challenging to implement, show significantly improved ability to provide useful inferences for the credit risk of firms in the South African and global contexts. The subsequent chapter outlines the methodology followed in order to estimate default probabilities for South African firms using the two most basic classes of the structural models.

CHAPTER 3
RESEARCH METHODOLOGY

3.1 INTRODUCTION

"*Complete realism is clearly unattainable, and the question whether a theory is realistic enough can only be settled only by seeing whether it yields predictions that are good enough for the purpose in hand or that are better than predictions from alternative theories."* (Friedman, 1953).

The insight provided by Friedman sets the background for the manner in which the research moves forward to evaluate whether the particular structural models are sufficient for the collective purpose(s) required of the structural models. This chapter outlines the methodology for estimating default probabilities of South African companies using South African financial statement and market data for the Merton and Delianedis & Geske models. The methodology regarding the estimation of the inputs required for each structural model's calculation of probability of default is discussed in detail, with reference to the theory of the structural models provided in the previous chapter.

3.2 DATA

3.2.1 Firm selection process

For the purpose of this paper, the top 22 companies of the Johannesburg Stock Exchange (JSE) in terms of market capital as listed by Bloomberg have been selected. The top 22 companies were chosen for simplicity and ease of access to data. Most empirical analyses regarding the performance of structural models in predicting default probability exclude financial companies as their capital structures are a lot more complex and further removed from the simplifying assumptions of zero coupon debt structure assumed by most structural models. However, the purpose of the research is to provide an idea as to whether structural models can provide insight to credit risks of companies, thus including financial firms in the sample allows for a broader picture to be painted in assessing the structural models performance.

Although a range of firm types are included in the top 22 of the JSE these firms are likely to be of similar investment credit grade. It should be noted that selecting the top 22 firms of the JSE as the sample of firms thus limits the ability to assess the performance of the structural models for a variety of credit grade firms. The sample of the 22 firms on the JSE with the largest market capital is also perhaps not most likely to represent firms that operate in markets with limited available data and the study could be significantly improved in this regard.

3.2.2 Market value of equity and equity volatility

The firms daily share price over a four-year period up until 31 December 2014 is recorded from Bloomberg in order to estimate the volatility of the equity. The GARCH (1,1) model is chosen to estimate the volatility of equity.

The GARCH (1,1) model is a mean reversion model and assumes that volatility is pulled back to its long-term average at a certain rate. The GARCH (1,1) model does not suffer from 'Ghost Feature' problems in volatility estimation and as such is expected to act as a reliable estimate for volatility of equity for the firm (Alexander, 2008). In the base-case scenario the volatility of the firm's equity is set to the unconditional volatility as calculated by the GARCH (1,1) model.

The market value of the firm's equity is taken as the share price as at 31 December of each year over the period 2009 to 2014. The market value of equity is taken as the value of a single share as opposed to market cap value. This is since solving non-linear simultaneous equations, required in the estimation procedures of firm value and volatility, prove to be computationally more efficient when per share values are used.

3.2.3 Expected rate of growth of the firm's value

The expected growth rate of the firm's value, μ, is estimated from the Capital Asset Pricing Model (CAPM). The CAPM estimates the required rate of return on equity by use of the following equation provided by Elton *et al.* (2011:287).

$$\bar{R}_i = R_F + \beta_i(\bar{R}_M - R_F) \tag{3.1}$$

Where:

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} \tag{3.2}$$

The CAPM thus estimates the required rate of return on the firm's equity by calculating the co-variance between the log returns of shares and the market as well as the variance of the log returns of the market as at 31 December in each period, where the expected growth rate of the firm's equity is being estimated. The variance and co-variance is calculated using the daily share prices of the firms and market over the prior four-year period as recorded by Bloomberg. The expected return on the market is calculated as the mean of the log returns of the JSE over the prior four-year period. The expected growth rate of the firm's value can then be estimated by de-levering the required return on equity (Sundaran and Das, 2010). The primary focus of the research is on risk-neutral default probabilities and the expected growth rate of the firm's value is thus simplified to be approximated by the required rate of return on equity as per the CAPM, where R_F is set as described in section 3.4 where the interest rate is set for the various models.

3.3 TIME HORIZON

3.3.1 Merton model time horizon

The Merton model assumes a single issue of outstanding zero-coupon debt; it is also common practice to assume that face value of the debt becomes due within a one-year time horizon as outlined in Wang and Suo (2006) along with Tudela and Young (2003).

The time to default is also extended to five-years to evaluate the Merton models ability to estimate longer-term probabilities of default. The Merton model(s) in this paper thus produce estimates for the probability that the firm will default in one years' time as well as in a five-year period in the extended time horizon case.

3.3.2 Delianedis-Geske model time horizon

The Delianedis-Geske (D&G) model, as described in the previous chapter, assumes two tranches of zero-coupon debt that become due in the short and long term respectively. In this paper is it assumed that the short-term debt becomes due within a one-year time horizon and the long-term debt becomes due over a five-year time horizon, as is common within credit risk literature and applications of the D&G model.

The D&G model(s) in this paper thus produces three different estimates of the probability of default for the firm. The first estimate produced is the short-term probability of default, which represents the probability of the firm defaulting within a one-year time horizon. The second estimate produced is the long-term probability of default, which represents the probability of defaulting over a five-year time horizon on condition that the firm has not defaulted at the end of the first year. The last estimate produced is the total probability of default, which represents the probability of defaulting at a one-year or five-year time horizon.

3.4 SETTING THE INTEREST RATE

Following a similar approach of Holman *et al.* (2011), the risk-free interest rate is substituted with a constant one-year forward financing rate as it can be assumed that the face value of outstanding debt will be serviced at this rate over the next year. In this paper, it is assumed that debt is financed at the one-year prime rate as financed at Rand Merchant Bank.

The prime rate is a worst-case scenario for the interest rate at which the debt will be serviced however; it is worth noting that this approach could be enhanced by considering various forward risk-free rates plus a spread.

Table 3.1: Prime Rates as at 14 March 2015

Prime financing rate 2009	0.15
Prime financing rate 2010	0.105
Prime financing rate 2011	0.09
Prime financing rate 2012	0.09
Prime financing rate 2013	0.085
Prime financing rate 2014	0.085

Source: Rand Merchant Bank

3.5 SETTING THE DEFAULT POINT

As described in the previous chapter the default point is the threshold point for the firm's value, which when crossed, triggers the default of the company on the debt outstanding. For each firm the short-term and long-term debts are extracted from the financial statements using Bloomberg. The short-term and long-term debt values used are those Bloomberg provides from financial statements of each firm. In this paper it is also further assumed that all debt financing is obtained in Rand value and interest payments are thus only made in Rands.

3.5.1 Merton model(s) default point

In the Merton model, the default point is defined as the face value of the zero-coupon bond due. For the purposes of this paper the face value of the zero-coupon bond is taken as the short term debt/liabilities as provided by the financial statement of position. This is then converted to debt outstanding per share by dividing by the number of shares outstanding for each firm. The default point is defined in terms of debt per share as this provides ease in calculation as well as greater convergence to a solution in solving simultaneous equations for firm value and volatility.

In the KMV approach, as described in the previous chapter, the face value of the zero-coupon debt due in one-year is calculated as the sum of the short-term liabilities and half of the non-current or long-term liabilities. A third adaptation of the Merton model is also applied which is named the 'total' style approach where the default point is set to the full sum of short term and long term liabilities as per the SOFP (statement of financial position).

3.5.2 Delianedis-Geske model(s) default point(s)

The D&G model provides two points of possible default, where the default points are the face values of the one-year and five-year zero-coupon outstanding debt issues respectively. The face values of the one-year and five-year zero-coupon outstanding debt issues are taken as the short and long-term debt values respectively as provided by Bloomberg. Once again, this is converted to debt per share and the debt per share is set as the default point.

For the ‘KMV styled’ adaptation of the D&G model in this paper, the default point for the one-year time horizon is set as the short-term debt per share plus half of the long term debt per share as recorded from the SOFP provided by Bloomberg.

3.6 SETTING THE BENCHMARKS

The one-year and five-year probability of default as measured by the Bloomberg model for the probability of default is set as the market expectation or reasonable probability of defaulting over the period for the firm. The outputs from the various models are compared to the Bloomberg estimate for the PD in order to assess the performance of the structural models.

3.7 DEFAULT PROBABILITY ESTIMATION

3.7.1 Merton model default probability

The risk-neutral probability of default is given by the following closed form expression as presented in the literature review:

$$N\left\{\frac{1}{\sigma\sqrt{T_1 - t}}\left[LN\left(\frac{D}{V_T}\right) - \left(r - \frac{1}{2}\sigma^2\right)(T-t)\right]\right\} \tag{3.3}$$

The symbols in the equation above have their usual meaning as described in the previous chapter. The firm value V_T and volatility of the firm σ are unobservable and are estimated by solving the following set of simultaneous equations provided by Sundaran and Das (2010).

$$E_T[V_T, \sigma] = V_T\, N(d) - e^{-r(T-t)}\, D\, N(d - \sigma\sqrt{T-t}) \tag{3.4}$$

$$\sigma_E = \sigma V_T \frac{N(d)}{E_T} \tag{3.5}$$

Where E_T and σ_E are the known share price of the firm and volatility of equity of the firm as calculated by the GARCH(1,1) model respectively. The data inputs as described in the methodology, are collected and read into statistical computer package, R (R Development Core Team, 2016), R is then used to solve the solve the simultaneous equations in order to calculate and estimate the probability of default as per the Merton model. The real-world probability is calculated similarly to the risk-neutral probability of default where r is replaced with μ (the expected growth rate of the firm’s equity) in equation (3.3). The R code used to implement the Merton model can be found in Appendix A.

The method makes use of the Newton Raphson algorithm to solve the non-linear simultaneous equations. The method requires initial guesses for the firm value V_T and volatility σ, the Newton Raphson method is very sensitive to these initial guesses.

In order to avoid the induction of negative firm volatility, the simultaneous equations were solved using solver in Excel initially and these solutions are then input as initial guesses for the parameters being estimated. Alternatively, to using Excel solver iteratively to find an initial solution, Crosbie and Bohn (2003) propose the following initial values for solving the system:

$$V_T = E_T + D_T \tag{3.6}$$

&

$$\sigma_V = \sigma_E \frac{E_T}{E_T + D_T} \tag{3.7}$$

3.7.2 Delianedis-Geske model default probability

The risk-neutral probabilities of default for the D&G model is given by the following expressions as presented in the previous chapter:

$$\text{risk neutral short run PD} = 1 - N(d_1) \tag{3.8}$$

$$\text{risk neutral long run PD} = 1 - \frac{N_2[d_1; d_2, \rho]}{N(d_1)} \tag{3.9}$$

$$\text{risk neutral total PD} = 1 - N_2[d_1; d_2; \rho] \tag{3.10}$$

Where the symbols have their usual meaning for the model as described in the literature review in the previous chapter. The real-world probabilities of default are obtained by substituting μ (the expected rate of growth of equity) for the risk-free rate (r) in equations (3.8), (3,9) and (3.10).

Once again estimates are required for V^*(cut-off value), V_T (firm value) and σ (volatility of firm value) in order to calculate the probabilities of default. Sunduran and Das (2010) provide the following system of non-linear equations to estimate unknown paramaters:

$$V^* = D_1 + B_{2,T_1} \tag{3.11}$$

$$E_T = V_T N_2\left[d_1 + \sigma\sqrt{T-t}; d_2 + \sigma\sqrt{T-t}; \rho\right] - D_2 e^{-r(T_2 - t)} N_2[d_1; d_2; \rho] - D_1 e^{-r(T_1 - t)} N(d_1) \tag{3.12}$$

$$\sigma_E = \sigma V_T \frac{N(d_1)}{E_T} \tag{3.13}$$

The system of non-linear simultaneous equations are solved in R by making use of the Newton Raphson algorithm, simirlarly to the Merton model estimation, Microsoft Excel solver was used to provide starting guesses for the three unkown paramaters to be estimated. The R-code used to estimate and implement the D&G model can be found in Appendix A.

It should be noted that the system of simultaneous equations only holds instantaneously as noted by Crosibe and Bohn (2003). This procedure can thus be improved upon by using the more complex iterative procedure outlined in Vasalou and Xing (2004).

3.8 SUMMARY

In short, the probability of default as determined by the Merton and D&G models with their required inputs and estimates are calculated for a sample of 22 firms over the period 2009 to 2014 and compared to the Bloomberg probability of default in order to assess the reasonability of the estimates provided by the structural models. Table 3.2 summarises the assumptions and inputs for the various modes. The results obtained from applying the outlined methodology and procedures along with the concurrent analysis of these results are presented in the next chapter.

Table 3.2: Summary of method inputs

	Merton	**Merton 'KMV'**	**Merton 'Total'**	**D&G**	**D&G 'KMV'**
Market value of equity.	Share price	Share price	Share price	Share price	Share price
Volatility of equity	Unconditional vol. Garch(1,1)	Unconditional vol. Garch(1,1)	Unconditional vol. Garch(1,1)	Unconditional vol. Garch(1,1)	Unconditional vol. Garch(1,1)
Expected firm value growth (μ)	Approximated by CAPM (r)	Approximated by CAPM (r)	Approximated by CAPM (r)	Approximated by CAPM (r)	Approximated by CAPM (r)
Interest rate	Prime rate	Prime rate	Prime rate	Prime rate	Prime rate
Default point	Current liabilities per share as per the SOFP	Current liabilities + half non-current liabilities per share as per SOFP	Total liabilities of firm per share as per SOFP	Current and non-current liabilities per share at each default point respectively as per SOFP	Current liabilities + half non-current liabilities per share and then non-current liabilities per share at second default point
Time Horizon	1 and 5 years independently	1 and 5 years independently	1 and 5 years independently	1 and 5 years for each default point respectively	1 and 5 years for each default point respectively
Estimating unobservable parameters	Newton-Raphson algorithm for non-linear simultaneous equations.	Newton-Raphson algorithm for non-linear simultaneous equations.	Newton-Raphson algorithm for non-linear simultaneous equations.	Newton-Raphson algorithm for non-linear system of equations.	Newton-Raphson algorithm for non-linear system of equations.

CHAPTER 4
RESULTS AND OBSERVATIONS

4.1 INTRODUCTION

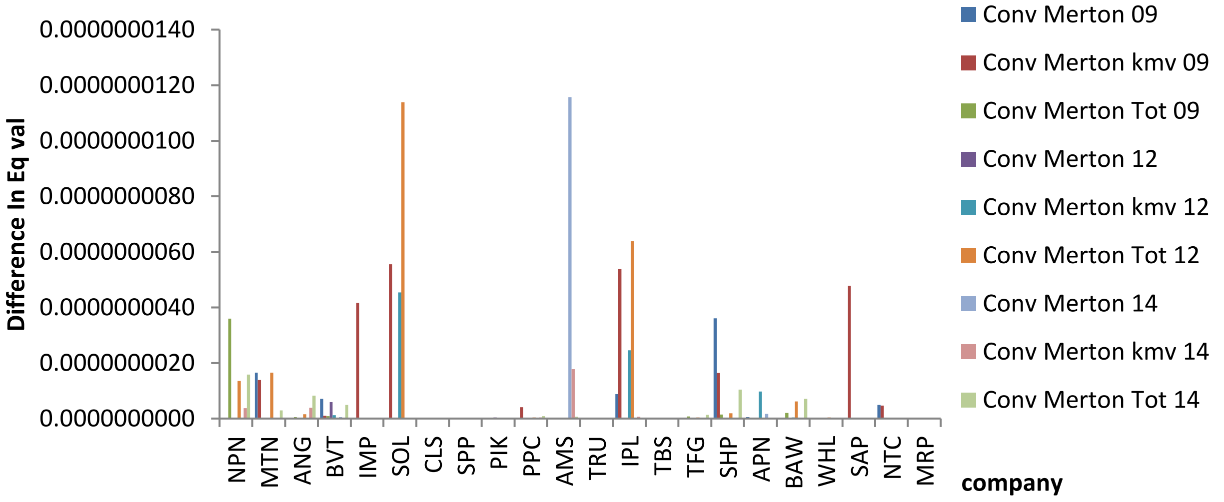
In this chapter the results and outcomes of applying the methodology described in Chapter 3 to estimate the probability of default from market and financial statement data are analyzed in detail. The probabilities of default are calculated for the 22 companies over the period 2009 to 2014 using the Merton and Delianedis & Geske class of stuctural models and compared to the Bloomberg probability of default in order to evaluate the estimation of the probability of default over time.

4.2 MERTON MODEL(S) ONE-YEAR PROBABILITY OF DEFAULT

4.2.1 Merton models firm value and firm volatility estimation

The ability of estimation procedure to succesfully solve the simultaneous equations required to estimate the value and volatility of the firm is measured by the output column 'convergence'. The convergence measure, measures the absolute difference between the values of the expected solution of the simultaneous equations and the solutions obtained when using the values from the Newton Raphson method. Figure 4.1 provides a graphic of the convergence for the various models. It is clear that all convergence values all extremely close to zero indicating that R was able to succesfully satisfy the conditions of the simultaneous equations and find accurate estimates for the firm value and volatility of the firm value according to the procedure used.

Figure 4.1: Convergence to solution 2009-2014 Merton models

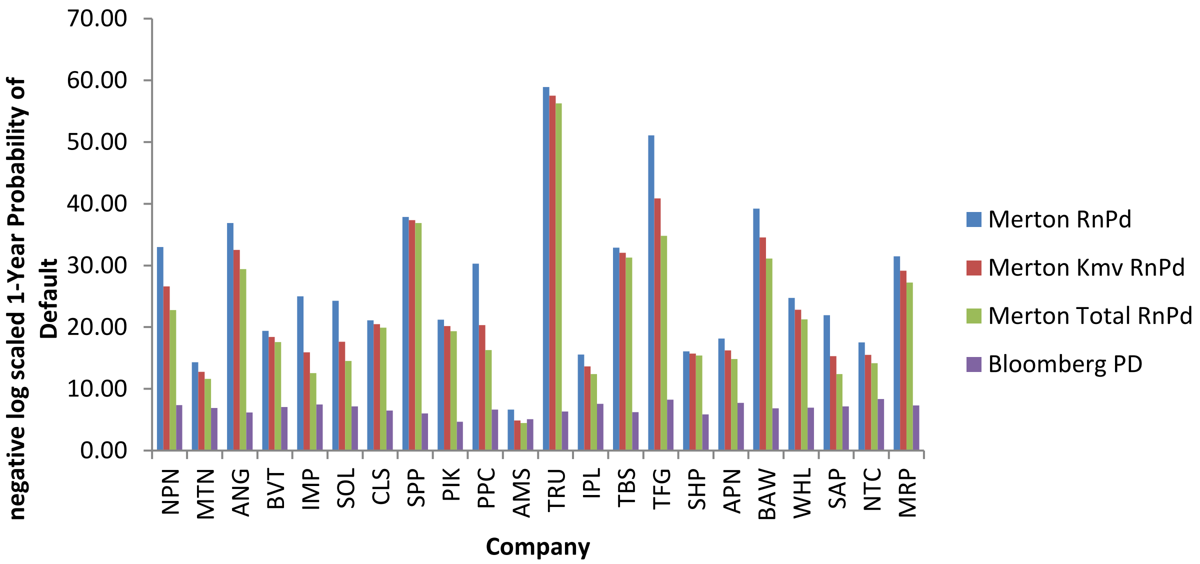


4.2.2 Merton models default probability

The results for estimating the probability of default over a one-year time horizon for 22 companies using the Merton model(s) for the years 2009 to 2014 can be found in figure 4.2 along with the corresponding graphics in Appendix B.

The Merton style models seemingly significantly under predicts the probability of default over a one-year time horizon, as all probabilities of default also close to zero with the exception of the company 'AMS'. Figure 4.2 includes only risk-neutral probabilities of defaults from the Merton models as the risk-neutral probabilities of default acts as an upper bound for the probability of default in comparison to the real-world probabilities of default as shown in the literature review in Chapter 2. Since default probabilities lie between zero and one with many firms yielding probabilities of default close to zero, a negative log scale is applied to the default probabilities in order to allow for an easier visual comparison. The larger values thus represent default probabilities that are closer to zero.

Figure 4.2 Merton model(s) results for 2009



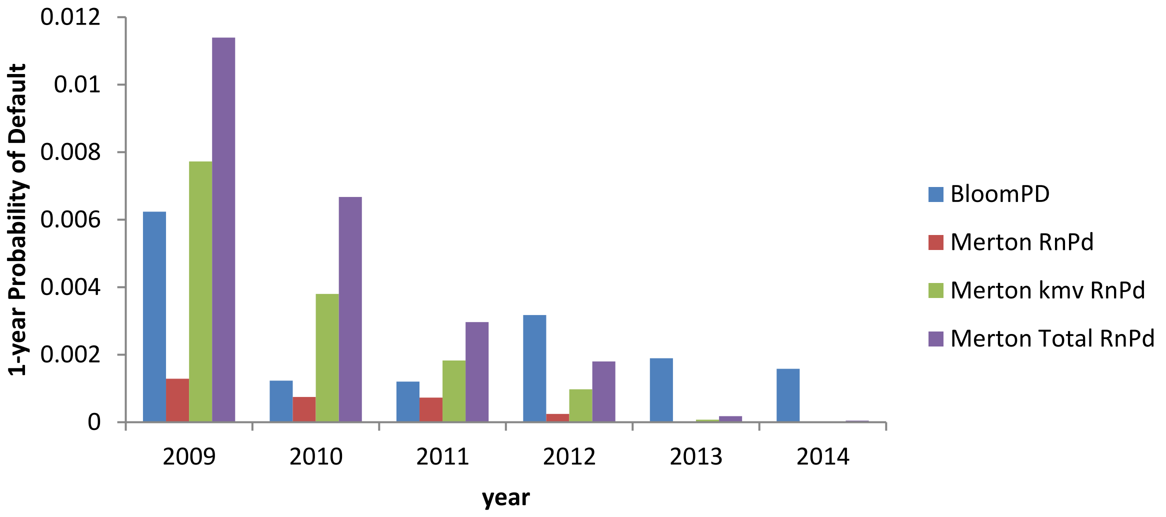
The significant under prediction of default probabilities as illustrated in Figure 4.2 appears to agree with empirical research that suggests the Merton models significantly under predict the probability of default for investment grade firms.

4.2.3 AMS results

The results across the three variations of the Merton model for the company AMS in particular are evaluated in order to identify why AMS was the only company for which the probabilities of default predicted by the models were significantly different to zero.

Figures 4.3 & 4.4 provide the probability of default along with the other paramaters estimated for the Merton models for the company AMS over the period 2009 to 2014. The corresponding table can be found in Appendix B.

Figure 4.3: Probability of default AMS Merton Models 2009-2014



The results provide some interesting insights into the workings of the Merton model and the two adaptations of the Merton model that have been applied. Firstly AMS is significantly different to other companies in terms of the leverage ratio, as measured by equation (2.9), produced by the three Merton models as this is significantly higher than that of other companies. This would suggest that the Merton model requires a significant amount of leveraging to be incorporated into the firm's capital structure in order to capture the credit risk associated with the firm. This agrees with the evidence found in the study of Holman *et al*. (2011) where the only firms that produced default probabilities significantly different to zero, were highly leveraged in comparison to the rest of the firms in the sample.

Figure 4.4: Firm leverage and volatility AMS Merton Models 2009-2014

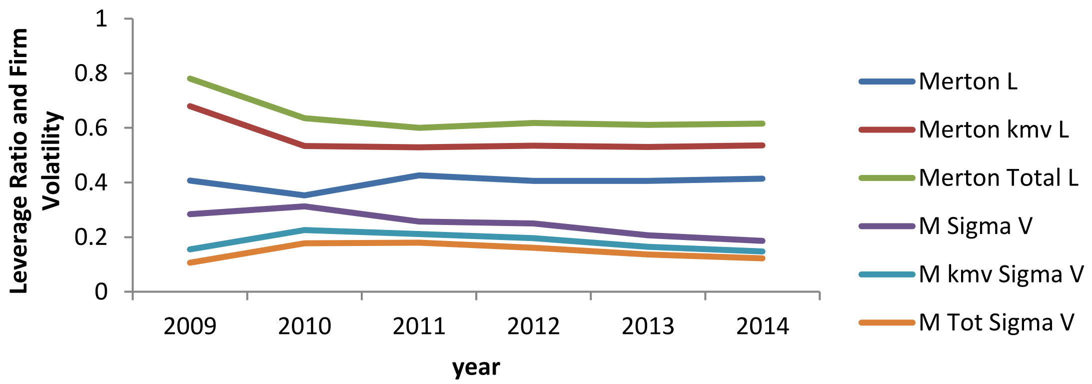


Figure 4.4 illustrates the effect of the choice of default point or Merton model adaptation in predicting the probability of default. As the face value of the debt outstanding is increased as in the KMV and total styled approaches, the firm's leveraged position increases while simultaneously decreasing the volatility of the firm's value.

Figure 4.3 suggests that the KMV styled approach provides the most consistent estimate for the probability of default when there is sufficient leverage and volatility for the Merton model to capture the probability of default. In the years 2013 and 2014, the probability of default estimated by the Merton models, is once again very close to zero which is not a very realistic or useful estimate. It appears that this drop off in ability to capture the probability of default coincides with a drop in the firm volatility.

4.2.4 Power of the Merton model

The Merton model estimates the risk-neutral probability of default by making use of the following equation:

$$N\left\{\frac{1}{\sigma\sqrt{T_1 - t}}\left[LN\left(\frac{D}{V_T}\right) - \left(r - \frac{1}{2}\sigma^2\right)(T - t)\right]\right\} \qquad (4.1)$$

The equation for the probability of default remains the same irrespective of the adaptation of the Merton model applied. The adaptation or form of Merton model applied thus changes the estimates for the value of the firm and hence leverage as well as the volatility of the firms value. The probability of default in the Merton models are thus determined by the estimates of the value of the firm V_T and the volatility of the firm's value σ. The probability of default for the Merton model is thus graphed as a function of the Leverage ratio as well as firm volatility value in order to determine combinations of these inputs for which the Merton Models produce estimates significantly different to zero. The R-code to produce these graphics can be found in Appendix A.

Figures 4.5 & 4.6 confirms the idea that the Merton models only captures the probability of default or credit risk associated with the firm in the presence of high leverage or firm volatility as calculated by the model. This explains why the Merton models predicted almost zero probability of default for all firms only excluding the highly leveraged AMS.

Furthermore, the low levels of debt financing amongst the top 22 South African firms as illustrated by the low leverage ratios in figure 4.5, supports the findings of Holman *et al*. (2011) that suggests sub-optimal capital structures may explain the inability of the Merton model to accurately capture the credit risk associated with the firm.

Figure 4.5: 2009 Level curve for Merton model PD less than 0.00001

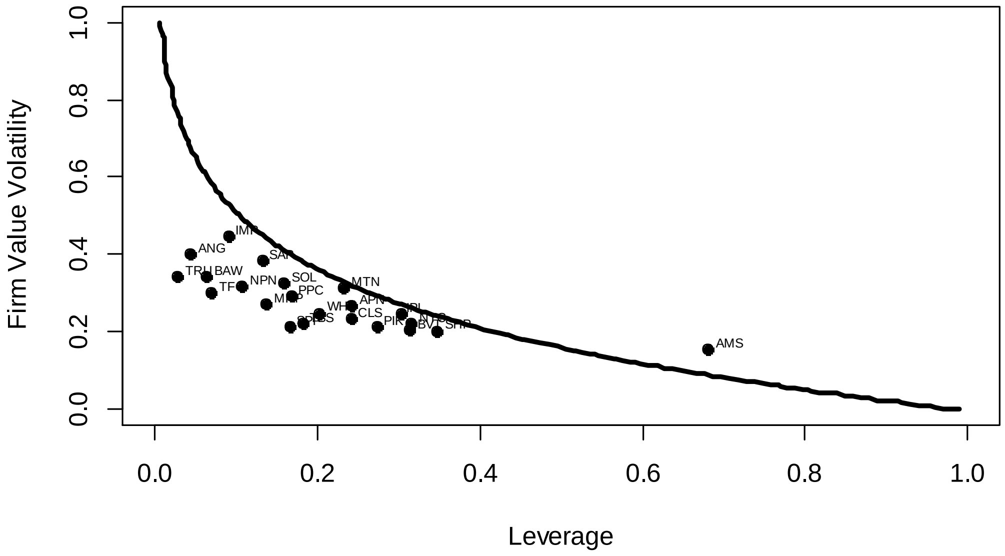
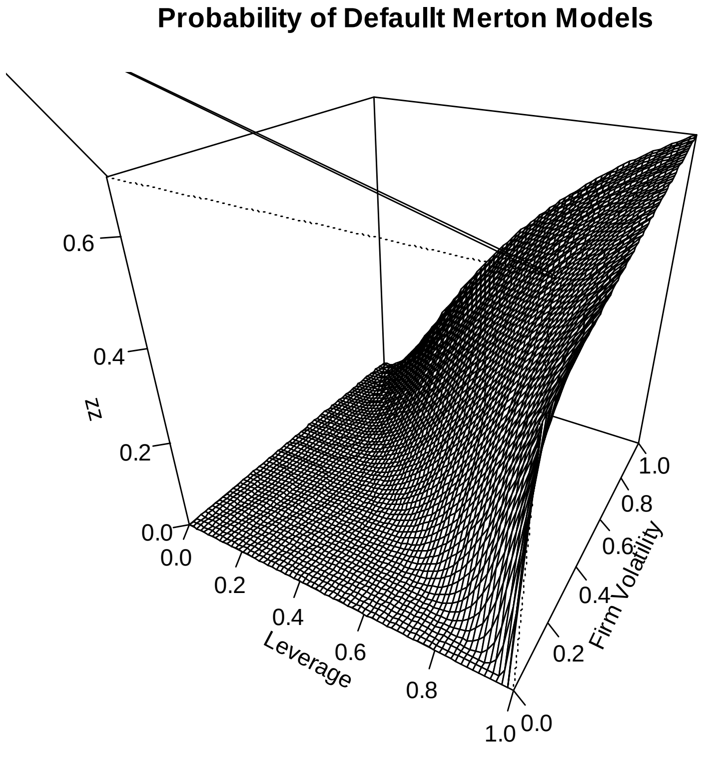
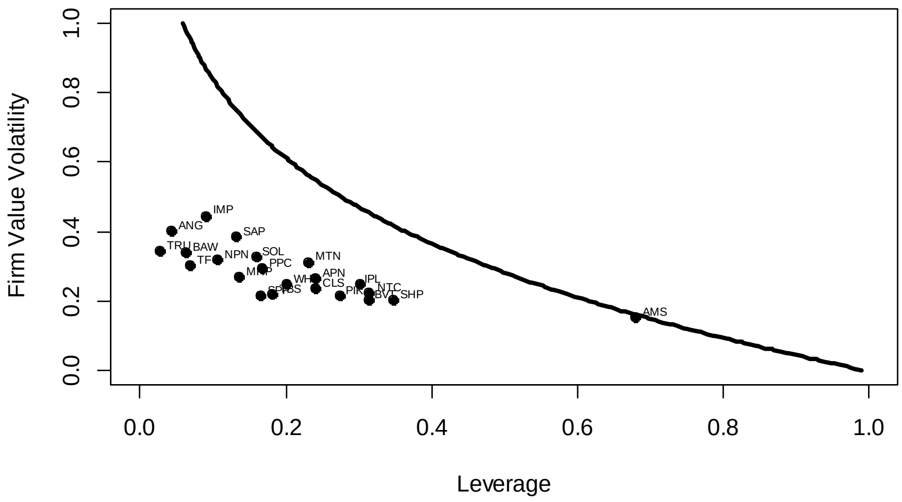


Figure 4.6: Power of the Merton Models



Figures 4.6 along with the two level curves also suggest that in the presence of an already large combination of leverage and firm value volatility, an aggresive trade-off for more leverage and less firm volatility as with using the total styled approach will lead to more volatile and aggresive estimates probability of default. The KMV styled approach thus is expected to produce the more consistent estimates of PD when there is sufficient leverage and volatility to capture the probability of default as it moderately increases leverage for a trade-off in firm volatility in the estimation procedure.

Figure 4.7: Level curve for Merton model PD less than 0.01



4.3 DELIANEDIS & GESKE(S) ONE-YEAR PROBABILITY OF DEFAULT RESULTS

4.3.1 D&G firm value, cut off value and firm volatility estimation

Figure 4.8: Convergence D&G models 2009

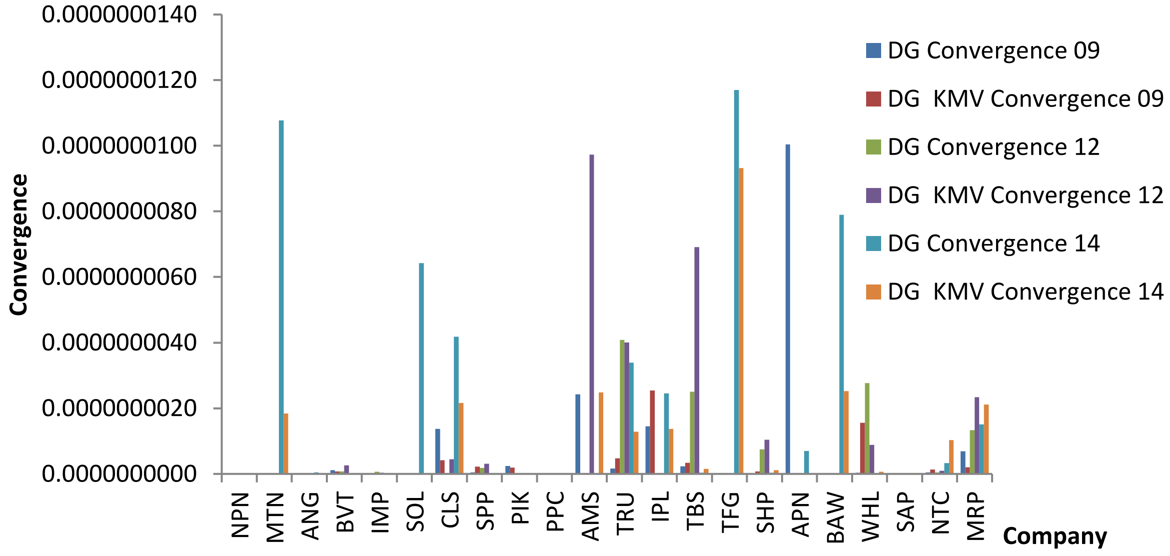


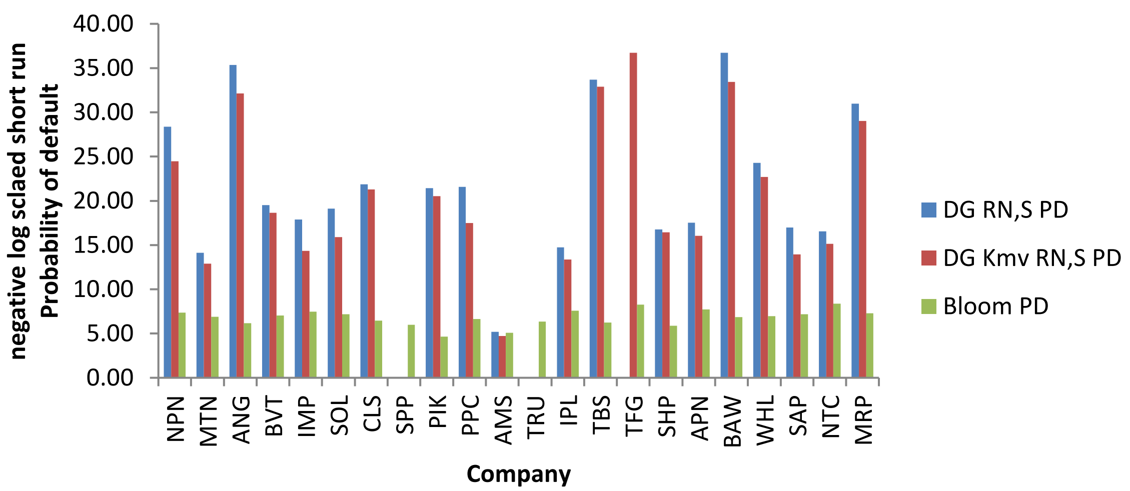
Figure 4.8 illustrates the ability of the Newton Raphson Method to solve the non-linear simultaneous equations required by the D&G models to estimate the probability of default. The convergence values are all close to zero indicating that the method was successful in finding solutions that satisfy the constraints given the starting guesses used by the model as discussed in chapter 3.

4.3.2 D&G short-term probability of default

The results for estimating the probability of default over a one-year time horizon for 22 companies using the Delianedis & Geske model(s) for the years 2009 to 2014 can be found in figure 4.9 along with the corresponding graphics in Appendix C.

The short run PD is once again transformed onto a negative log scale to allow for more demonstrative visual comparison. The D&G models appear to similarly predict close to zero and in some instance exactly zero for short run default probabilities of all firms except for AMS. The significantly large negative log values in figure 4.9 illustrates the under prediction of default probabilities from the D&G model as these values correspond to default probabilities that are close to zero.

Figure 4.9: Delianedis & Geske 'short run' PD 2009



4.3.3 D&G short-term probability of default AMS results

The results across the two variations of the D&G model for the company AMS in particular are evaluated in order to identify why AMS was the only company for which the probabilities of default predicted by the models were significantly different to zero. Figures 4.10 & 4.11. provides the probability of default along with the other paramaters estimated for the D&G models for the company AMS over the period 2009 to 2014. The corresponding table is found in Appendix C.

The firm has significantly larger leverage ratios than other companies in conjunction with a large equity volatility. This suggests that the D&G model similarly is only able to reasonably predict the probability of default when the firm is highly leveraged and the value of the firm is also highly volatile. This can be seen as the estimates of the probability of default move towards zero when the level of volatility of firm value drops for a given level of leverage as can be seen in the years 2012, 2013 and 2014.

Figure 4.10: Delianedis & Geske 'short run' PD AMS

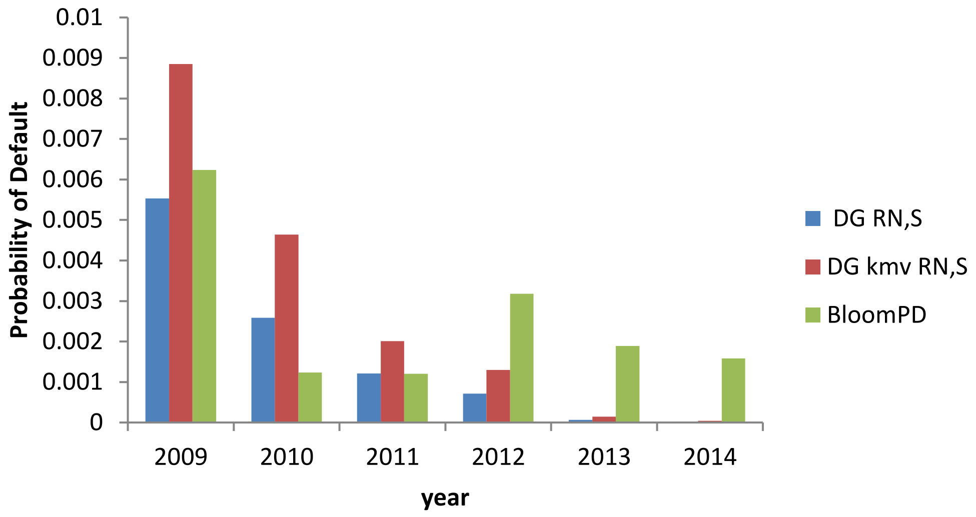


Figure 4.11: Delianedis & Geske leverage and firm value volatility AMS

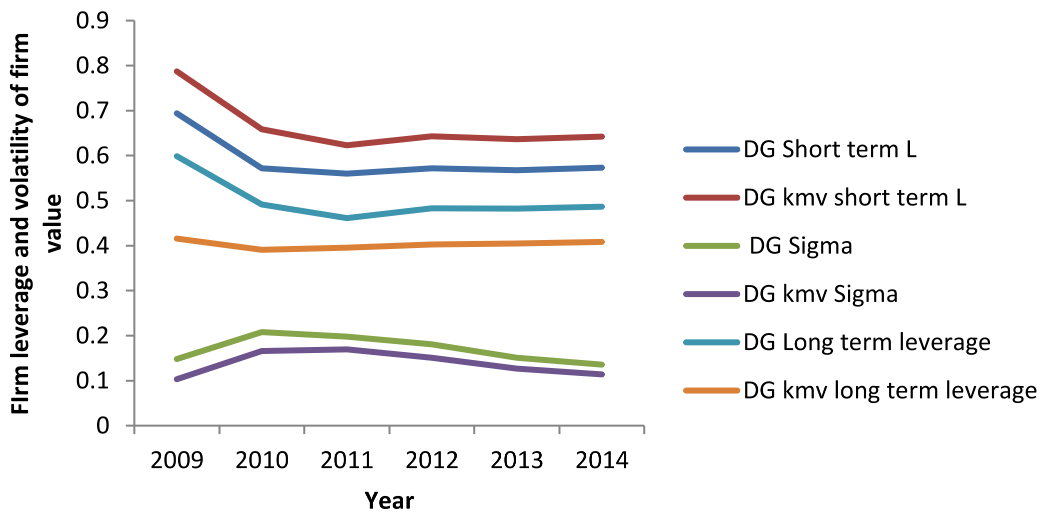


Figure 4.11 illustrates the effect of the choice of the face value of the short term debt due in the D&G model. The KMV style adaptation applied increases the value of the short term debt due as a function of the long term debt due of the firm.

This increase results in a larger short term leverage ratio in the estimation procedure while simultaneously aggressively decreasing the long term leveraged position and volatility of the firm value. Figure 4.11 shows that the KMV adaptation increases both the critical cut off value (V^*) as well as the value of the firm (V_T) in the estimation procedure. It appears that the KMV adaptation overly aggressively increases the short term leveraged position when the firm is sufficiently leveraged and is thus likely to overestimate the short term probability of default as evident in figure 4.10.

The D&G model produces an estimate for the risk-neutral short run probability of default by means of the following equation:

$$\text{risk neutral short run PD} = 1 - N(d_1) \tag{4.2}$$

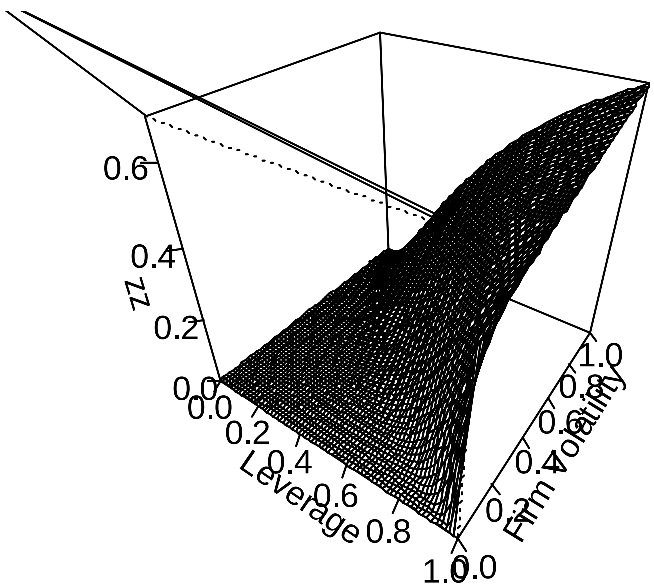
Where:

$$d_1 = \frac{Ln\left(\frac{V_T}{V*}\right) + (r + \frac{1}{2}\sigma^2)(T_1 - t)}{\sigma\sqrt{(T_1 - t)}}$$

Similar to the analysis of Merton this function is graphed as a function of short term financial leverage and firm value volatility in order to provide insight into the circumstances under which the D&G models provide useful estimates. The short term financial leverage is measured as:

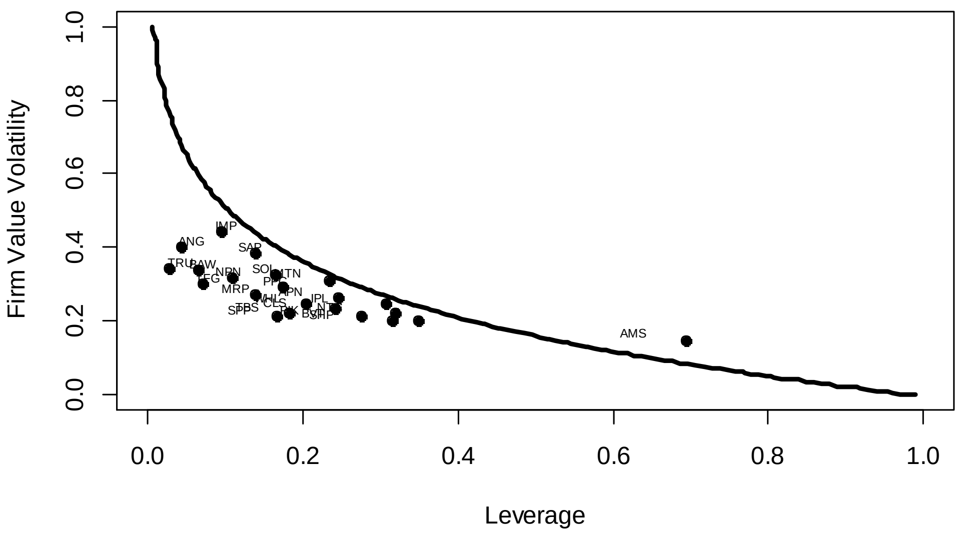
$$L = \frac{e^{-r(T_1-t)}V^*}{V_T} \tag{4.3}$$

Figure 4.12: Power of the D&G models



Analogous to the Merton model, the D&G model requires a relatively large combination of short-term leverage and volatility in order to produce meaningful estimates of the short run probability of default. Figure 4.13 confirms that AMS is the only firm with sufficient short-term leverage and volatility for the D&G model to produce an estimate significantly different to zero.

Figure 4.13: 2009 Level curve for D&G model PD less than 0.00001

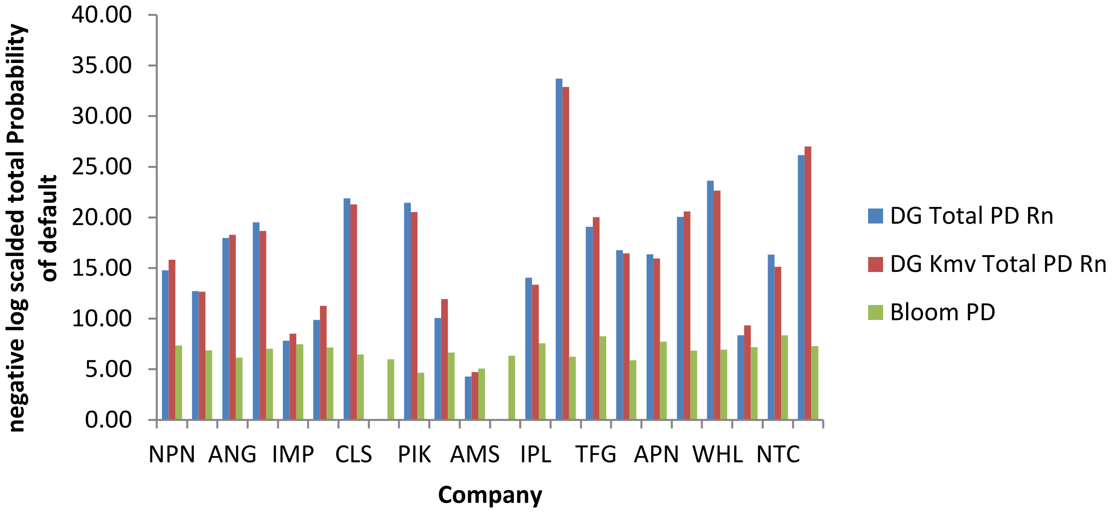


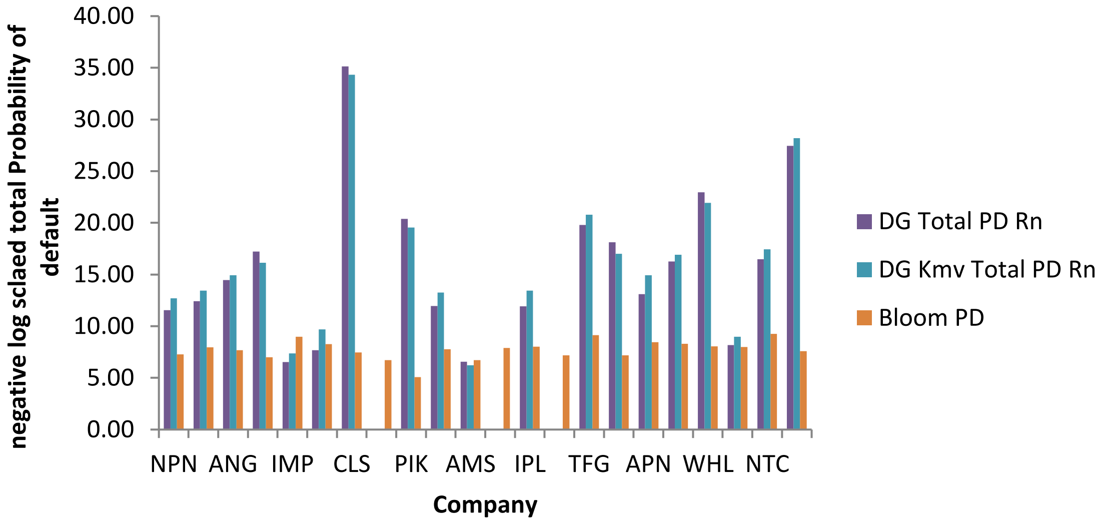
4.3.4 D&G total probability of default

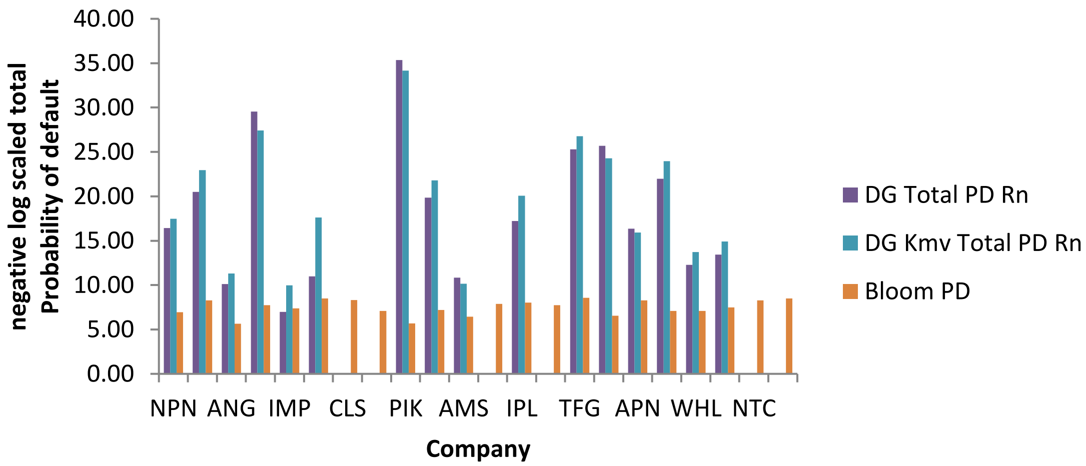
The D&G model calculates a total probability of default based on the short-term and long-term probability of default. The total probability of default produced by the D&G models is then compared to the Bloomberg one-year probability of default in order to evaluate whether the total probability of default can provide a more reasonable estimate of the short-term probability of default. Figure 4.14 illustrates the estimation of the probability of default for the sample years 2009, 2011 and 2014 in order to view the estimation of the probability of default over time. .

The default probabilities in figure 4.14 are scaled onto a negative log axis in order to allow for visual comparison. Figure 4.14 below indicates that the D&G total probability of default significantly improve the estimation of the probability of default, as the majority of the probabilities are considerably further away from zero. However, the total probability of default as determined by the D&G model(s) cripplingly under predicts the probability of default and yields predictions or estimates of zero and close to zero in many instances.

Figure 4.14: Delianedis & Geske total probability of default 2009, 2011 & 2014

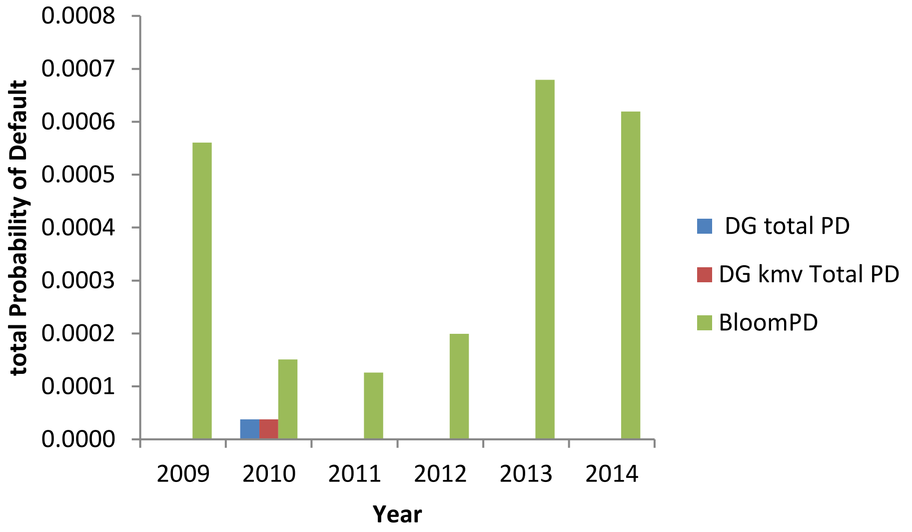


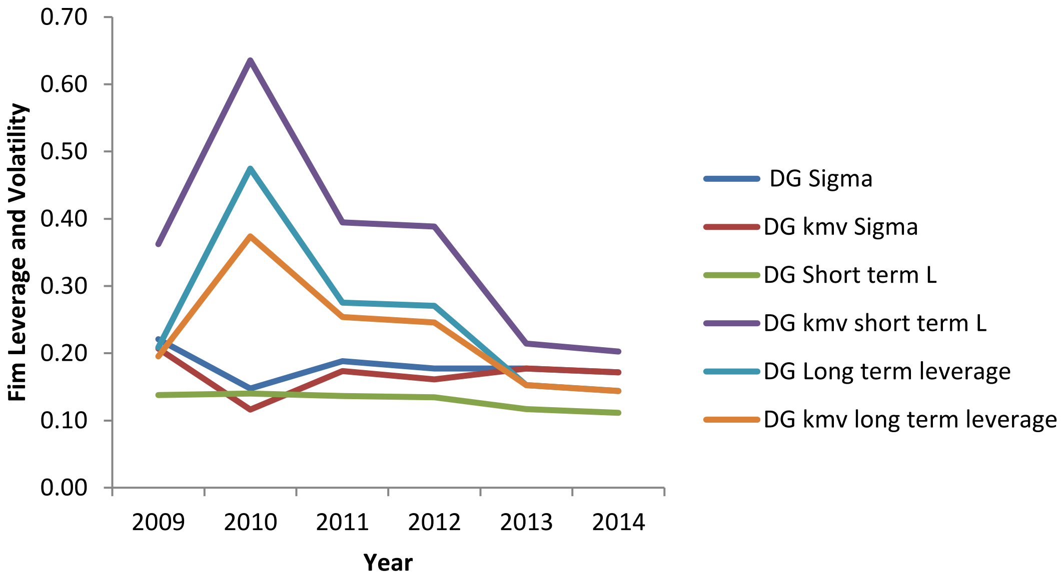




More encouragingly there now appears to be five firms, namely AMS, IMP, NTC, SOL and SAP, for which the D&G total probability provides useful estimates of the one-year probability of default in certain instances. The characteristics of these firms are analysed in detail in order to gain insight into the conditions under which the total probability of default as per the D&G model provides useful estimates.

Figure 4.15: Delianedis & Geske 'total' PD NTC

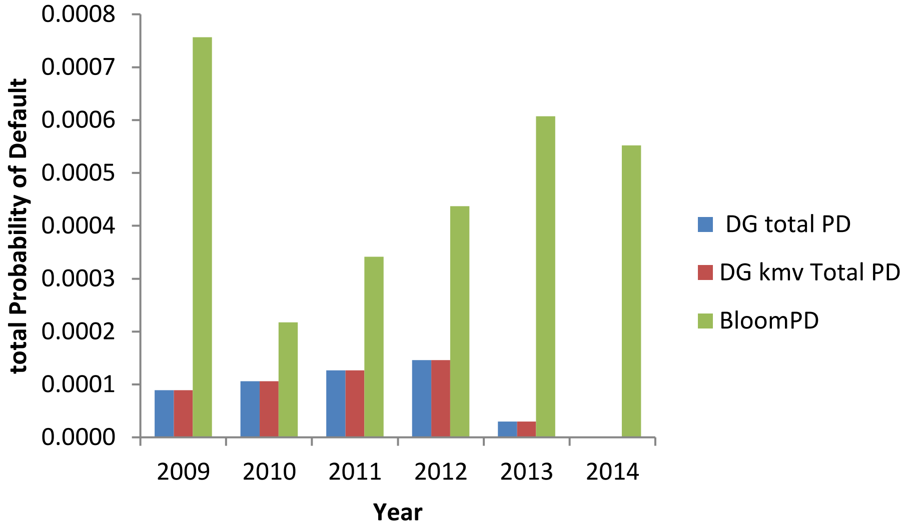




Evaluating the conditions under which the D&G total probability of default provides useful estimates is not as simple as in the previous cases, as the total probability of default is a function of both the short and long-term probability of default.

Figure 4.15 provides a useful starting place as the model only provides a reasonable estimate in the year 2010. The Firm NTC has relatively low volatility and short-term leverage however, there is a large spike in the long term leverage ratio, indicating that the long term leverage ratio is potentially a big driver in generating total probabilities of default.

Figure 4.16: Delianedis & Geske 'total' PD SAP



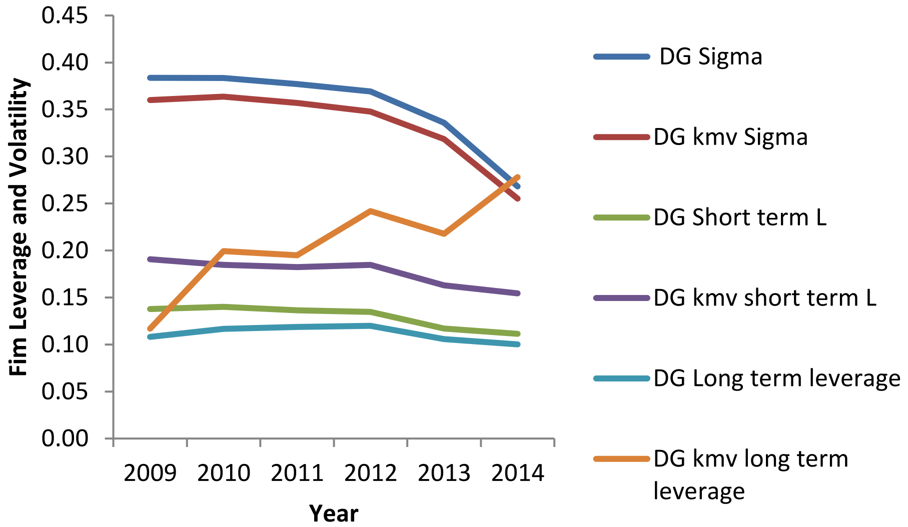
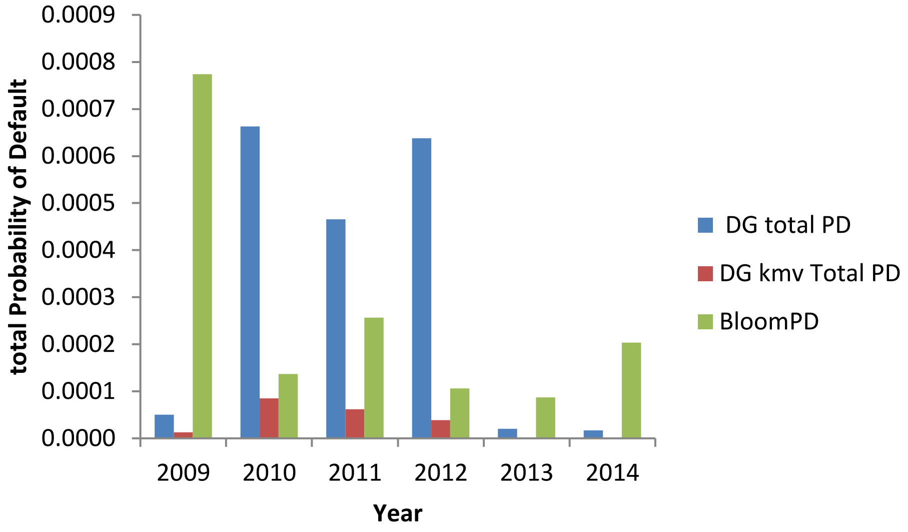


Figure 4.16 interestingly shows the scenario where the total probability of default appears to be driven by the volatility of the firm primarily as opposed to the leveraged position of the firm. It appears that the total probability of default produces estimates significantly different to zero in the case of significantly large volatility of the value of the firm in conjunction with a leveraged position.

Figure 4.17: Delianedis & Geske 'total' PD SOL



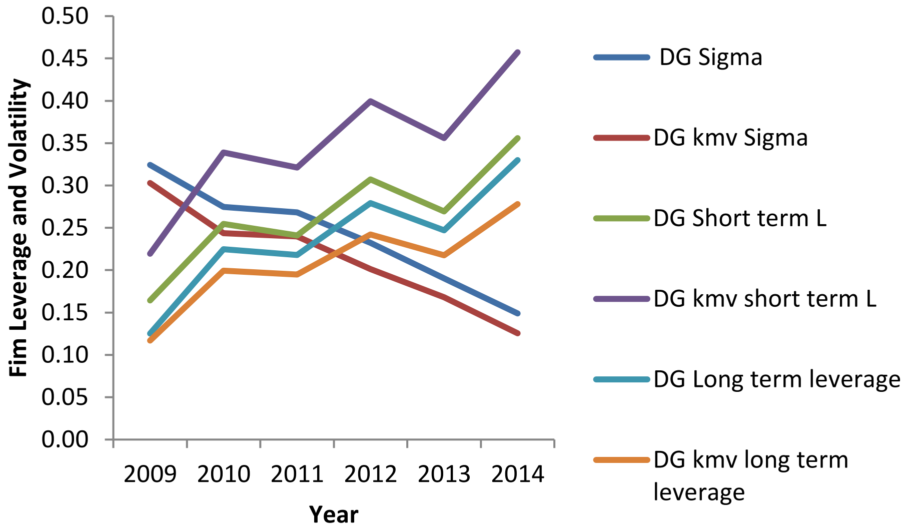
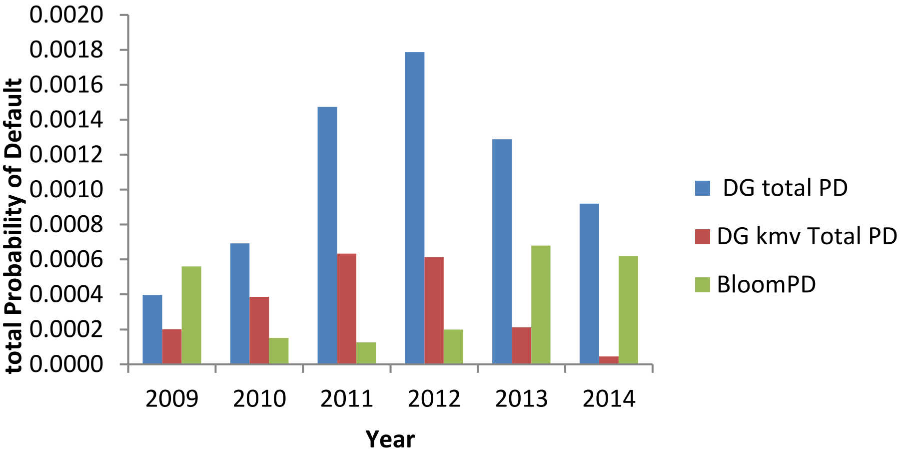


Figure 4.17 provides very useful insight into the conditions under which the total probability of default provides meaningful estimates. The short term and long-term leveraged positions are sufficiently large along with large volatility, the total default probability tends towards zero where the volatility of the firm drops below the critical region which is considered to be sufficiently large. The KMV adaptation of the model consistently under predicts the probability of default in comparison to the standard D&G model. The KMV adaptation produces significantly higher short-term leverage ratios however, simultaneously decreases the long term leverage and volatility suggesting that the long term leverage ratio and volatility are the key generators of the total probability of default.

Figure 4.18: Delianedis & Geske 'total' PD IMP



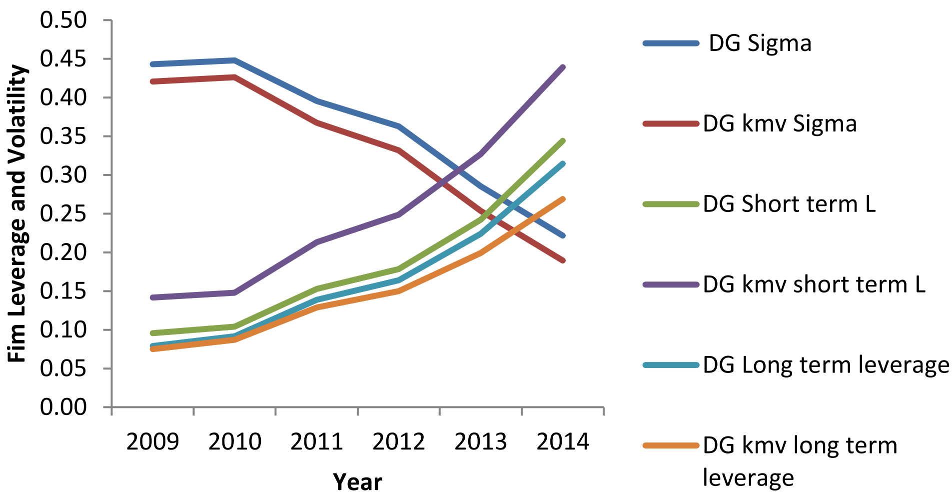


Figure 4.18 confirms that the total probability produces useful estimates where there is a combination of large firm value volatility as well as long-term leverage ratio. Examining the firms in the sample it is evident that most firms have moderate firm value volatility but significantly low leverage ratios and thus the model produces estimates of the probability of default that are close to zero in these instances.

Comparing figures 4.15 to 4.18 against 4.7 and 4.8, it is evident that the total probability of default produces reasonable estimates for a greater combination of volatility and leverage than the Merton model which requires considerably larger combinations of the two in order to produce probabilities of default significantly different to zero. The total probability of default appears to respond only marginally more reasonably to levels of leverage and volatility thus explaining why it was able to produce meaningful estimates for five of the firms as opposed to the single meaningful estimate in the case of Merton with the extremely leveraged AMS.

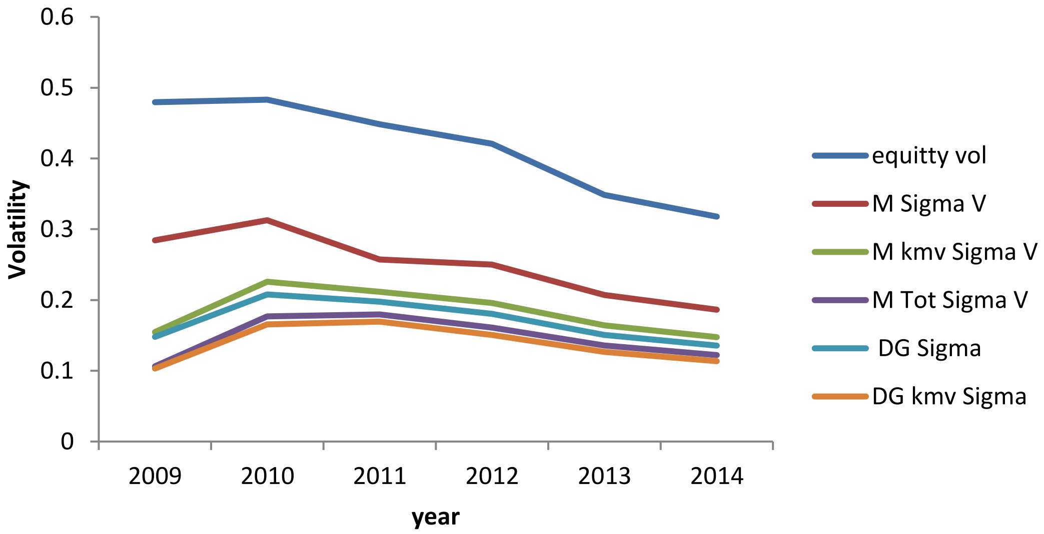
4.4 COMBINED RESULTS FOR AMS

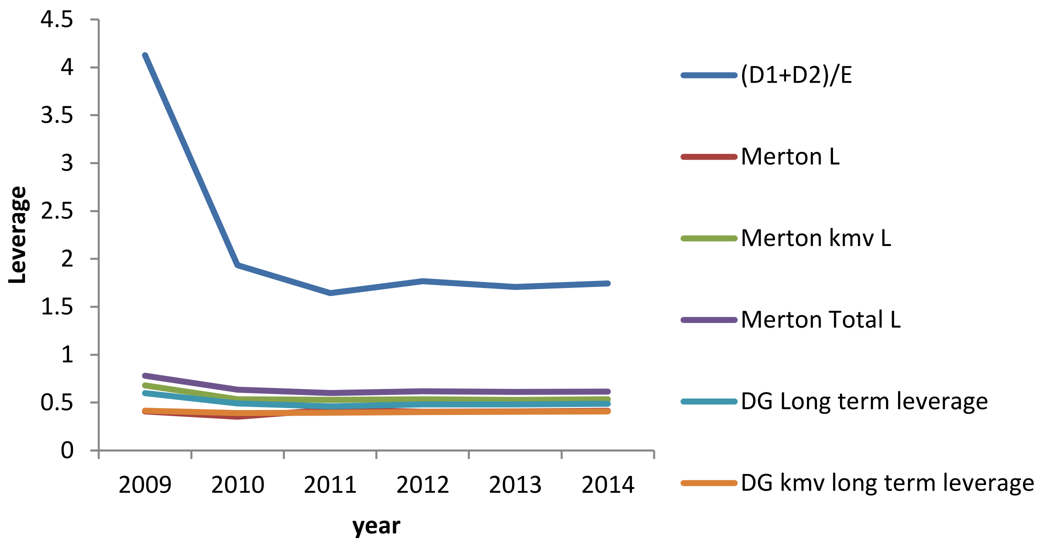
It has been comprehensively shown thus far that the Merton as well as D&G class of structural models only produces meaningful estimates for high levels of leverage for the firm as well as large levels of firm value volatility. The capital structure and firm equity volatility for AMS is now analysed in order to provide insight into why this particular debt structure and equity volatility was the only one in the group of firms to produce sufficient leveraged positions and firm value volatility in order for the structural models to produce meaningful estimates.

The results suggest that higher levels of equity volatility along with a debt structure where the value of the debt outstanding is significantly large in comparison to the value of equity, produces a leveraged position and firm volatility sufficiently large enough for the structural models to produce meaningful estimates. This can be seen as AMS is the only firm with debt value greater than the value of the equity, all other firms have debt values that are relatively small in comparison to the value of equity and thus do not produce the sufficiently large enough leveraged positions required by the structural models.

The equity volatilty of AMS is also significantly greater than all other firms thus indicating that a significantly large equity volatility produces the firm volatility which is large enough for the structural models to meaningfully estimate the probability of default. Figure 4.19 reaffirms this idea as when the equity volatility drops to the average level of the other firms in 2013 and 2014 the firm value volatility falls below the critical region for which structural models produce meaningful estimates.

Figure 4.19: Firm value volatilty and leverage vs debt structure and equity volatility





Thus in order for the structural models to provide meaningful or useful estimates for the probability of default, the firm being evaluated must exhibit a high level of equity volatility and posses a capital structure that incorporates a considerable amount of leverage. Under those conditions it appears that the structural models are able to reasonably estimate the probability of default.

The various structural models estimated probabilities of default is compared to the Bloomberg probability of default, in the instance where the firm displays sufficient leverage and firm value volatility for the structural models to applied, as is the case with the firm AMS for the period 2009 to 2012. The mean absolute difference between the estimated PD and the Bloomberg PD is used as a measure in an attempt to identify which structural model most accurately predicts the PD when there is sufficient leverage and volatility present.

Figure 4.20 risk-neutral probability of default estimation AMS

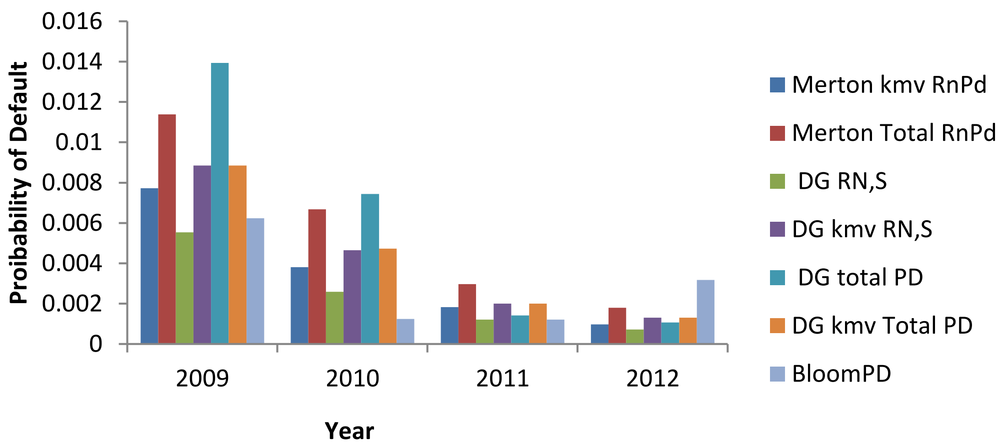


Figure 4.21 Real-world probability of default estimation AMS

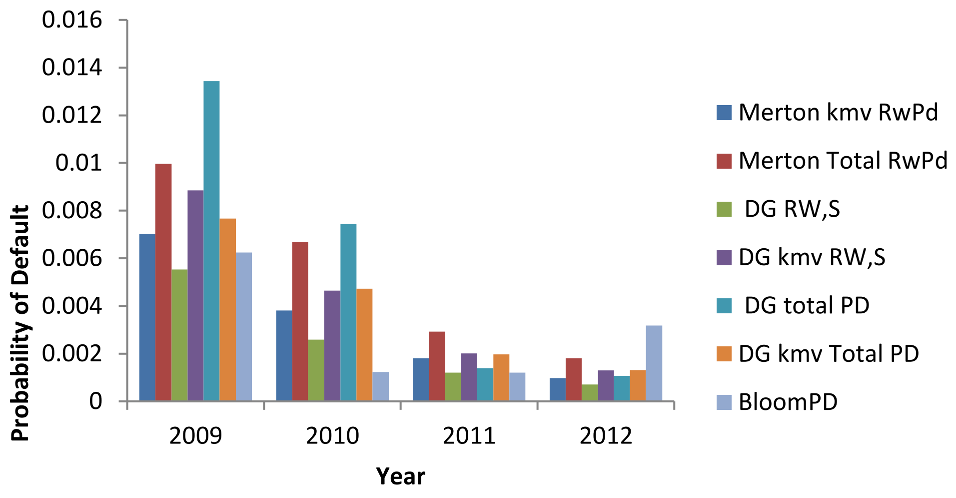


Figure 4.22 Mean absolute error of estimation across models for AMS

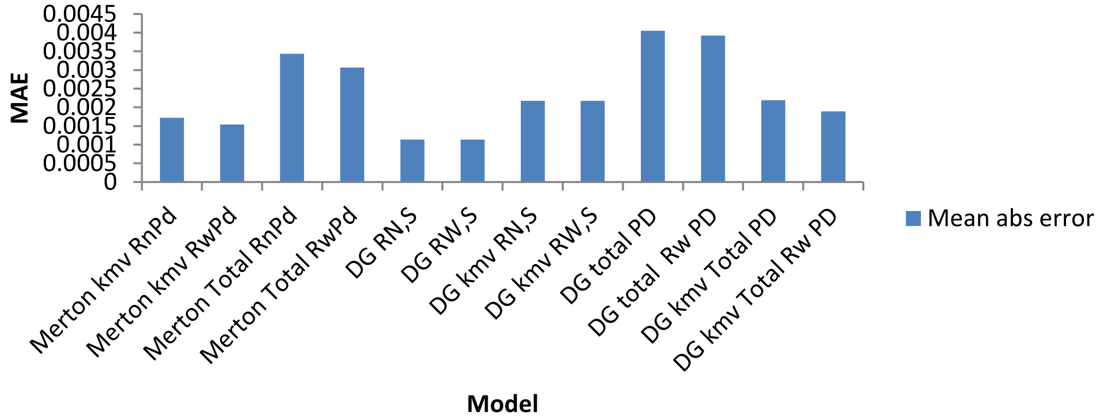
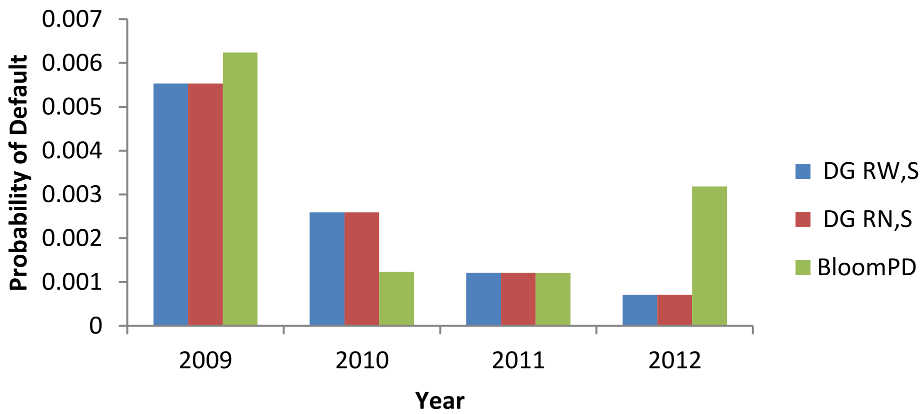


Figure 4.23 D&G short run risk-neutral probability of default estimates for AMS



The results indicate that the D&G risk-neutral short run probability of default may provide the most consistent estimate between the structural models in the instance where there is sufficient financial leverage and volatility for the models to produce meaningful estimates. The D&G risk-neutral short run probability of default has the lowest mean absolute error and appears to provide an estimate that closely matches the Bloomberg PD as seen in figure 4.22 and 4.23. However, the sample size of only four-years or four estimates is far too small to make any statistically significant claims around the performance between the structural models and warrants further investigation.

4.5 FIVE-YEAR PROBABILITY OF DEFAULT

The time horizon for default is extended to five years for the structural models and the results are compared to the Bloomberg five-year PD to assess whether the structural models can be used to provide estimates of longer-term probabilities of default.

4.5.1 Merton models probability of default

The results for estimating the probability of default over a five-year time horizon for 22 companies using the Merton model(s) for the years 2009 to 2014 can be found in figure 4.24 along with the corresponding graphics in Appendix B. The default probabilities are once again transformed onto a negative log scale throughout.

The Merton models perform significantly better over the extended five-year time horizon as the models produce meaningful estimates for a far larger number of firms in the sample. However, the problem remains that in multiple instances the Merton models produces probabilities of default that are close to zero which are clearly unrealistic and significantly under predict the probability of default of the firm.

Figure 4.24 Merton models five-year PD vs Bloomberg PD 2009

Extending the time horizon produces meaningful estimates for a far wider range of firm leverage and volatility combinations as expected from option pricing theory. Figures 4.25 and 4.26 confirm the Merton models ability to capture the probability of default for a much greater combination of leverage and volatility than the instance of one-year probability of default estimates.

Figure 4.26 suggests that the low levels of leverage amongst the firms is responsible for the Merton models producing estimates of close to zero for the five year probability of default for certain firms once again in agreement with the work of Holman *et al.* (2011). This suggests that the Merton models may provide a useful estimate for longer-term probabilities of default where the firm possesses a capital structure that incorporates a moderate to significant amount of leverage.

Figure 4.25 Merton model five-year probability of default

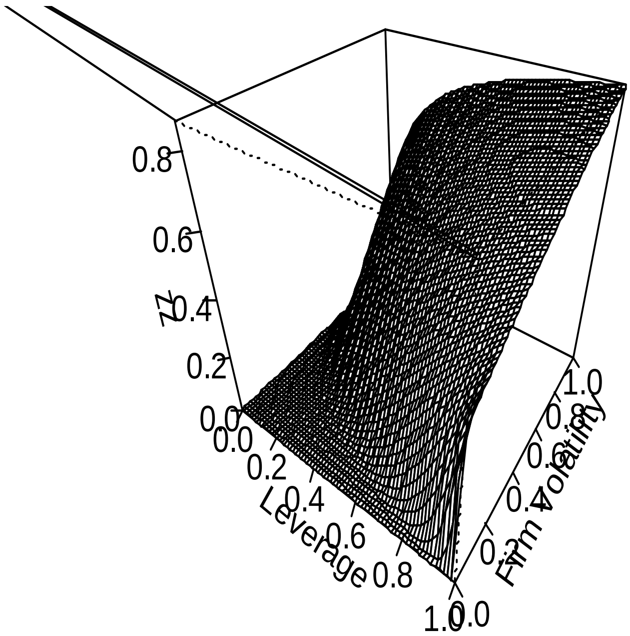


Figure 4.26 2009 Merton model level curve 5-year PD less than 0.0001

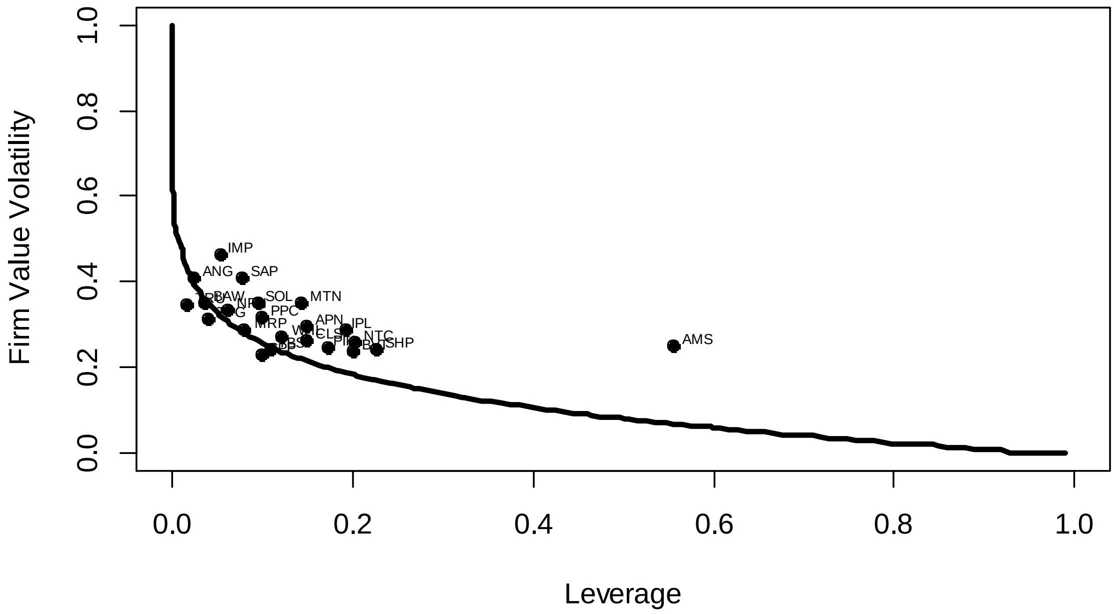
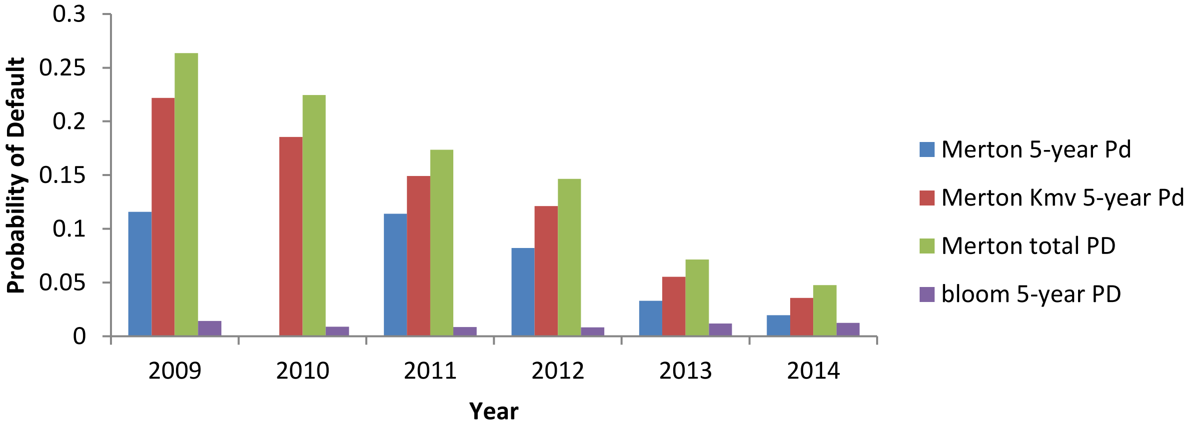


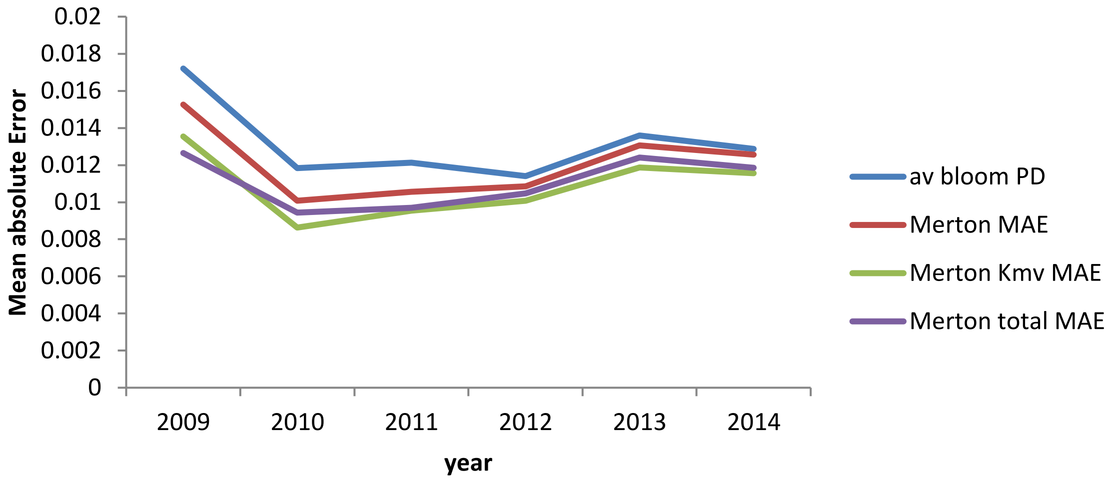
Figure 4.26 indicates that the firm AMS is significantly different to the other firms in the sample in terms of leverage. The five-year probabilities of default for the Merton models for AMS are found in figure 4.27 and provide an indication of the Merton models performance in the scenario of a highly leveraged firm. In the instance of the highly leveraged and volatile firm the Merton models appear to significantly over predict the probability of default.

Figure 4.27 Merton model results for AMS 5-year PD vs Bloomberg PD



The firm AMS is removed from the sample as an outlier and the mean absolute error of prediction for the Merton models is calculated and can be found in Figure 4.28. The results suggest that the KMV adaptation of the Merton model provides the better estimate for the five-year probability of default between the Merton models. Figure 4.24 shows the base Merton approach consistently underestimates the PD while the 'total' approach tends to over estimate the PD. Thus the KMV adaptation of the Merton Model allows for the most consistent capital structure assumption that produces firm leverage and volatility estimates for which the Merton model most accurately estimates the five-year probability of default.

Figure 4.28 Merton models mean absolute erorr vs Bloomberg PD



4.5.2 Delianedis & Geske models long-term default probability

The long-term probability of default from the D&G model provides the probability of defaulting in five-year's time, on condition of not having defaulted on the short-term debt at the end of the first year. The total probability of default is greater than the long-term risk-neutral probability of default in all instances as expected from equation 4.5 and 4.6. Thus, the total probability of default is used as the measure for the five-year probability of default produced by D&G model.

The analysis is thus analogous to that of section 4.3.4 presented previously in the results. The D&G total probability of default only provides meaningful estimates in five instances as previously explored as opposed to the Merton models which provide meaningful estimates in a far larger number of firms. The Merton models appear to react more reasonably to leverage ratios and volatility in the extended time horizon as opposed to the D&G model, evidenced by the far greater number of firms for which meaningful estimates were produced.

The D&G model produces the three different estimates for the probability of default by means of the following equations where the symbols have their usual meaning as defined in chapter 2.

$$\text{risk neutral short run PD} = 1 - N(d_1) \tag{4.4}$$

$$\text{risk neutral long run PD} = 1 - \frac{N_2[d_1; d_2, \rho]}{N(d_1)} \tag{4.5}$$

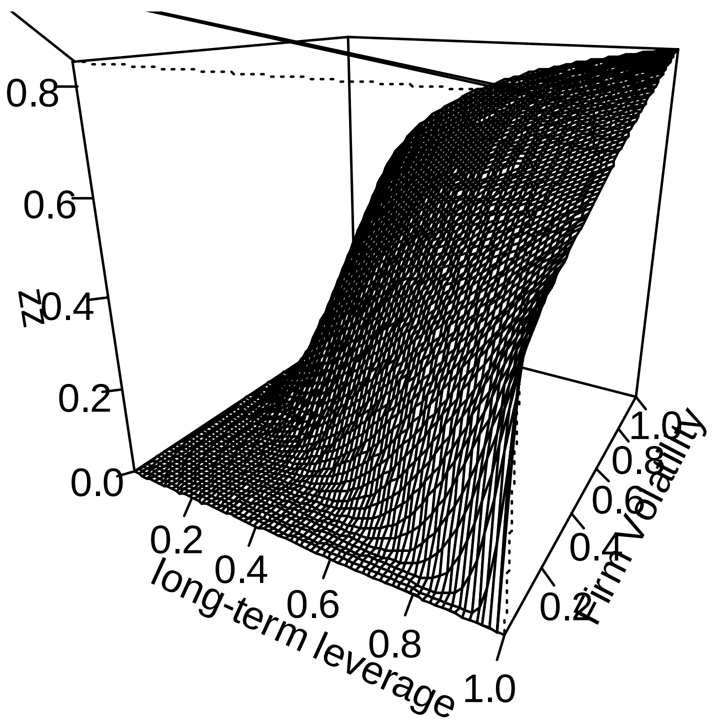
$$\text{risk neutral total PD} = 1 - N_2[d_1; d_2; \rho] \tag{4.6}$$

The analysis in section 4.3.4 suggested that the long-term leverage and volatility are the key drivers in producing risk-neutral total probability of default as per the D&G model. The risk-neutral total PD is graphed as a function of long-term leverage and firm volatility for a given level of short-term leverage in figure 4.29. Where long-term leverage is measured as:

$$L = \frac{e^{-r(T_1 - t)} D_2}{V_T} \tag{4.7}$$

The given level of short-term leverage is taken as 0.2 which represents the average in the sample of firms in 2009.

Figure 4.29 D&G total default probabilities and level curve



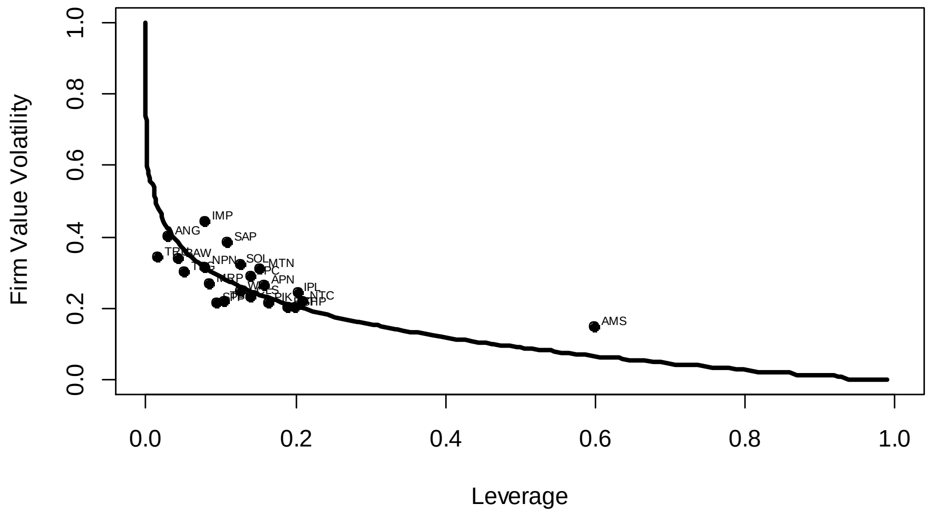


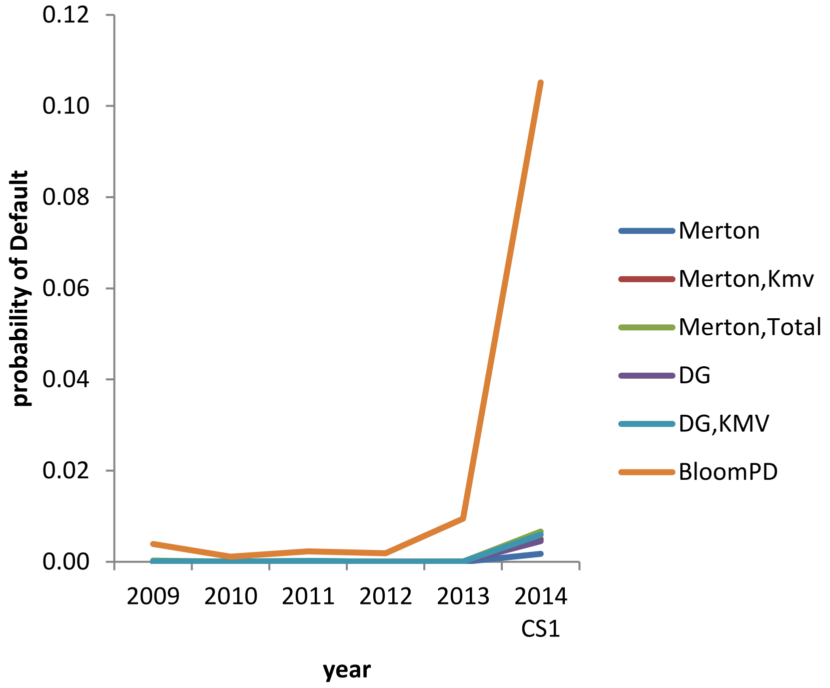
Figure 4.29 confirms that the total probability of default as measured by the D&G model requires large levels of long-term leverage and volatility in order to produce estimates of default probabilities significantly different to zero. Figure 4.24 and 4.26 along with figure 4.29 also confirms that the Merton model five-year PD responds more reasonably to combinations of leverage ratios and volatility as well as produces estimates that are a lot further away from zero than the D&G total probability of default. This would appear to suggest that the Merton model framework is preferred over the D&G model for evaluating longer-term default probabilities.

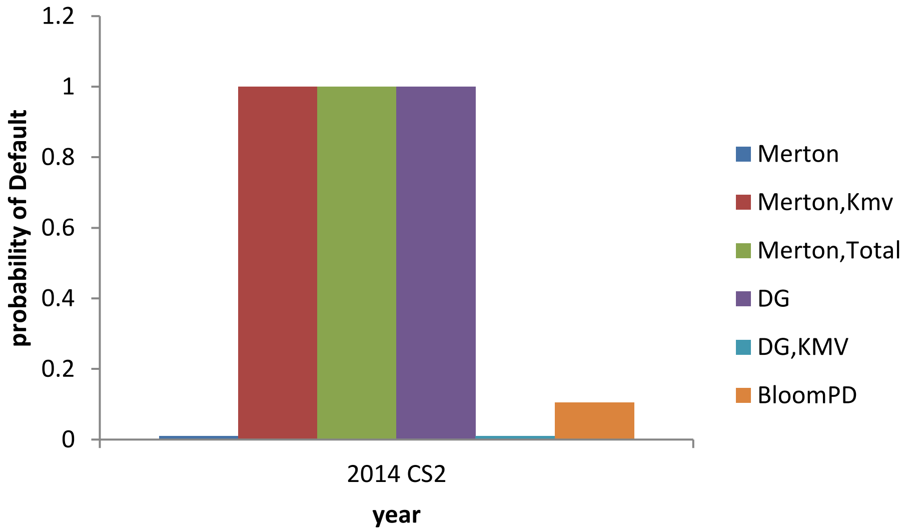
However, it should be noted that the KMV adaptation of the Merton model allows for an aggregation of short-term and long-term debt, since the firms in the sample are lowly leveraged this method is expected to produce more reasonable estimates than the D&G model where the debt classes are treated separately. This means that the D&G model may still be able to provide very useful information regarding the short and long-term credit risks associated with the firm, provided the firm incorporates a sufficient amount of debt financing into the capital structure and exhibits a volatile equity price.

4.6 AFRICAN BANK CASE STUDY

African Bank Limited (Abil) was one of the more recent significant financial failures of a major firm in the South African market. In August 2014, African Bank collapsed in the wake of a mountain of bad debt, forcing the government to intervene and appoint curators to oversee the restructuring of the company. (Fin 24, 2014). Abil thus provides an opportunity to assess how well the structural models of Merton and Delianedis & Geske are able to predict or forewarn of the deterioration of Abil's financial position in the build up to the collapse.

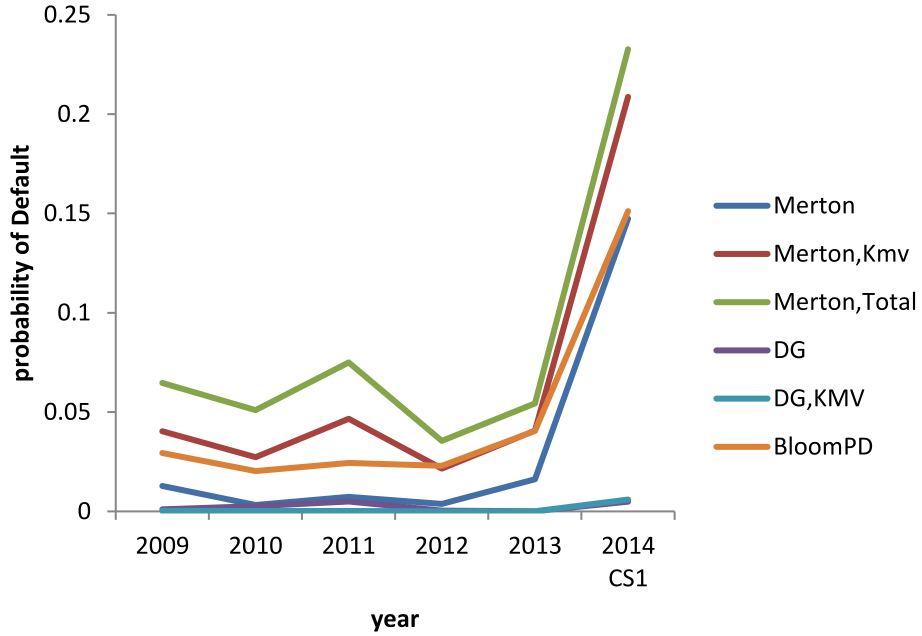
Figure 4.30 Abil one-year probability of defualt 2009-2014 CS1 &CS2

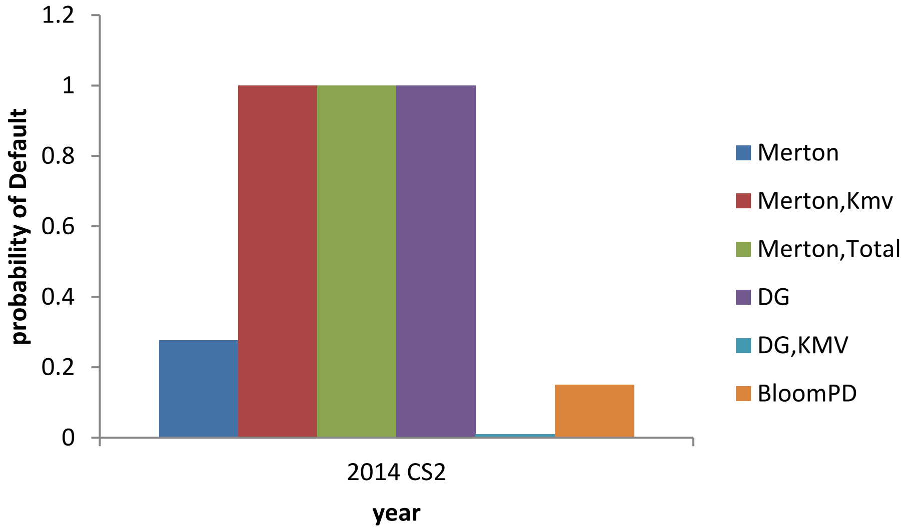




Abil's financial position as well as credit risk appears to be stable until 2014 in which it defaults. For this reason the results for both the one-year and five-year probabilities of default are split into two separate graphics for the periods 2009 up until the end of the first quarter of 2014 (CS1) and the second quarter of 2014 (CS2).

Figure 4.31 Abil five-year probability of default 2009-2014 CS1 &CS2





The rapid deterioration of Abil's credit worthiness as well as financial stability within 2014 suggests that probabilities of default require to be reassessed more frequently than an annualised period in order to be successful in effectively predicting forthcoming default.

The one-year probability of default as determined by the models of Merton and D&G significantly under predict the forthcoming financial distress of Abil up until the end of the first quarter of 2014 as shown in figure 4.30.

More encouragingly, the five-year KMV adaptation of the Merton Model appears to provide the best indication of the credit risk of the firm and forthcoming default. The model provides stable estimates as tracked by Bloomberg over the period of financial stability for Abil. However, it also sharply increases at the end of 2013 and the first quarter of 2014 most effectively reflecting the increased risk of default for Abil as illustrated in figure 4.31. This is in agreement with the works of Tudela and Young (2003) that suggests the Merton model provides a strong signal of forthcoming default or financial distress of a firm.

The second quarter of 2014 provides very interesting results as in both the one-year and five year estimates of the probability of default, the Newton Rapshon method was unable to find convergence to a solution for solving the unknown parameters of the sets of simultaneous equations in the Merton KMV and total approach as well as the D&G model. The estimates produced by these three models were thus used with the solutions found that were closest to fitting the constraints. These three models predict a probability of default of one at the end of the second quarter of 2014 successfully recognizing the forthcoming default of the firm although the parameters used did not fully satisfy the constraints of the system of equations specified in the estimation procedure.

In the case of the base Merton model as well as KMV styled D&G model where convergence to solutions were found for the second quarter of 2014, the models significantly under predicted or failed to accurately reflect the increased risk associated with Abil. The KMV adaptation of the D&G model adversely affects estimation procedure of the unknown parameters and even induces negative asset volatilities in some instances. The D&G model already incorporates the effect of the two tranches of debt in the capital structure and an aggregation of debt within either tranches thus adversely affects the estimation procedure.

The results of the second quarter of 2014 suggest that the method of solving simultaneous equations to solve for the unknown firm value and volatility is responsible for the weak performance of the structural models. This is illustrated by the inability of the method to obtain convergence to a solution in the period prior to default as well as the extreme manner in which it trades leverage for volatility in the estimation process where convergence was obtained. This is in agreement with the works of Crosbie and Bohn (2003) which highlight the weaknesses of this method to solve for unknown firm parameters.

Furthermore, this may suggest that the theoretical basis for the structural models may provide a useful framework for estimating the probability of default however, the method in which the unknown parameters of firm value and volatility were estimated contributed to the poor performance of structural models ability to estimate the probability of default.

4.7 SUMMARY

Overall, the Merton and D&G models produce default probabilities of close to zero in many instances that significantly under predict the credit risk or probability of default associated with the majority of the firms in the sample of the top 22 of the JSE. However, it has been shown that in the presence of sufficient financial leverage and volatility of the firms value, the structural models may provide useful credit risk information.

Furthermore, the low levels of debt financing and hence financial leverage amongst the top 22 firms of the JSE is largely responsible for the poor performance of the structural models. The procedure of solving simultaneous equations in order to estimate the unknown firm value and volatility are also shown to unfavourably affect the structural models performance.

The performance of the structural models is also significantly improved when considering extended time horizons for the probability of default and the models appear to respond to much wider combinations of leverage ratios and volatility. Extending the time horizon for default however does not solve the reoccurring problem that the structural models face in their inability to capture the credit risks associated with lowly leveraged firms. Extending the time horizon also appears to overstate the credit risk associated with firms that are highly leveraged.

Most encouragingly though, using Abil as a case study, the Merton model appears to be able to predict financial distress and when evaluated on a frequent to regular basis can provide good warning signs of forthcoming default or financial difficulty.

In the following chapter the research is viewed as whole; the ability of the research to answer the posed research question and objectives is evaluated. The scope and limitations of the research is also presented along with recommendations for potential further research.

CHAPTER 5
SUMMARY, CONCLUSION AND RECOMMENDATIONS

In the wake of the 2007 credit crisis, credit risk and the management thereof has become an ever-increasingly important field. Default and failure of financial firms have widespread economic impact and global regulatory authorities have stressed the need for incorporating counterparty credit risk (CCR) in derivative positions as well as increasing financial reporting standards to quantify CCR in derivative transactions in the form of CVA.

Default probabilities were shown to be essential to quantifying CVA as well as credit risk and of course provide a predictor of default. The literature review considered the various methods in which default probabilities can be determined and found that default probabilities from bond or CDS spreads obtained from the market was preferred for CVA quantification by the IFRS. CVA is intended to be a market-based measure and thus risk-neutral probabilities from structural models may theoretically be used for CVA quantification where bond or CDS data is not available.

The methodology then outlined the procedure followed to implement the two most basic classes of structural models in order to determine whether the structural approach could provide a simplistic alternative to modelling default where bond and CDS data is not available. The Merton and D&G models were shown to provide very limited information regarding the credit risk of the top 22 firms of the JSE, as the models produced estimates for default probabilities close to zero in many instances in line with previous empirical studies in the global and South African market.

The low levels of leverage amongst the South African firms were found to be responsible for the poor performance of the structural models. However, the Merton and D&G models may provide very useful estimates of default probabilities where the firm is sufficiently leveraged and has a highly volatile equity value.

This would suggest that the Merton and D&G models provide a partial solution to measuring CCR where the counterparty being evaluated operates in markets with limited credit market data. The term sufficiently leveraged is used loosely throughout the research and warrants further investigation into under what exact levels of leverage and volatility the Merton and D&G models can be used to provide a simplistic alternative to modelling default of firms.

One of the notable limitations in the approach followed in this research is the over simplification in the choice of sample firms. The top 22 firms of the JSE are not likely to represent firms that operate in market conditions with limited credit information. This limits the ability to evaluate the structural approach's capacity to provide a viable simplistic alternative to measuring CCR for firms that operate in such conditions.

Moreover, the problem still remains that the simplistic Merton and D&G models fail to capture the credit risks associated with the firm where the firm does not incorporate large amounts of leverage into its capital structure. Increasing the scope of the solution to include more complex alternatives to modelling default just as the literature review indicates, that the mathematically more complex extensions of the Merton framework may provide a viable alternative to modelling default and credit risk.

A suggestion for further research would thus be whether the inclusion of more complex models and methods of estimation in the structural approach are able to provide consistent and reliable estimates of default probabilities for South African firms from financial statement and market data. Such extensions include the incorporation of stochastic interest rates and first passage models such as the model of Leland and Toft (1996) and more complex estimation procedures of estimation as suggested in Vassalou and Xing (2004).

It is interesting to note that, although the simplistic class of structural models provides limited credit risk information, the Merton model still provides a good indication or prediction of forthcoming financial distress. This perhaps warrants further investigation into which of the structural models and perhaps methods of determining default probabilities, provide the best indication of forthcoming default by evaluating previous cases of default.

Overall, South African financial statement and equity market data can be used to provide insight to credit risk and the measurement of default probabilities of South African firms. More particularly the extensions of the Merton model are expected to provide more useful information regarding the credit risk associated with the firm.

REFERENCES

Alexander, C., 2008, *Market Risk Analysis Volume II: Practical Financial Econometrics*. New York: John Wiley & Sons.

Altman, E. I., 1989. Measuring Corporate Bond Mortality and Performance. *The Journal of Finance*, 44(4), September: 902-922.

Amato, J. D., Remolana, E. M. 2003. The Credit Spread Puzzle. *BIS Quarterly Review*, 5, December: 51-63.

Anderson, R. and Sundaresan, S. 1996. Design and valuation of debt contracts. *Review of Fiancial Studies*, 9, 37-68.

Arora, N., Bohn, J. R., Zhu, F. 2005. *Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models*. [San Francisco]: Moody's KMV.

Black, F., and Cox, J. 1976. Valuing corporate securities: Some effects of bond indenture provisions. *Journal of Finance*, 31, 351-367.

Bloomberg. (2012) *Bloomberg Professional*. [Online]. Available at: Subscription Service (Accessed: 03 April 2015).

Bloomberg Treasury and Risk Management.,2015, *Leveraging The Credit Valuation Adjustment To Improve Transparency For Treasurers*. [Online]. Available: http://www.bloomberg.com/professional/blog/leveraging-credit-valuation-adjustment-improve-transparency-treasurers/. [2015, April 03].

Byestrom, H., (2006) Merton Unravelled: A Flexible Way of Modelling Default Risk. *Journal of Alternative Investments,* Spring, 8(4), pp.39-47.

Chen, R. 2013. On the Geske Compound Option Model When Interest Rates Change Randomly-With an Application to Credit Risk Modeling. Working Paper. New York: Fordham University.

Collin-Dufresne, P., and Goldstein, R. 2001. Do Credit Spreads Reflect Stationary Leverage Ratios? *Journal of Finance*, 56, 129-157.

Crosbie, P., & Bohn, J. (2003). Modelling Default Risk. *Moody's* KMV Working Paper.

Delianedis, G., Geske, R. 1998. Credit Risk and Risk Neutral Default Probabilities: Information about Rating Migrations and Defaults. Working Paper. Los Angeles, California: Anderson School, University of California.

Delianedis, G., Geske, R. 2001.The Components of corporate credit spreads: Default, Recovery, Tax, Jumps, Liquidity and market Factors. Working Paper. Los Angeles, California: Anderson School, University of California.

Duan, J., C. 1994. Maximum Likelhood Estimation Using Price Data of Derivative Contract, *Mathematical Finance*, 4, 155-167.

Duffee, G.R. 1996. *Treasury yields and corporate bond yield spreads: An empirical analysis*. [Washington]: Federal Reserve Board.

Duffie, D. 1998. Credit Swap Valuation. Stanford, California: Graduate School of Business, Stanford University.

Duffie, D., Singleton, K., J. 1999. Modeling Term Structures of Defaultable Bonds. *The Review of Financial Studies*, 12(4), 687-720.

Elton, E.J., Gruber, M.J., Agrawal, D., Mann, C. 2001. Explaining the Rate Spread on Corporate Bonds. *The Journal of Finance,* 56(1), February:247-277.

Elton, E., Gruber, M., Brown, S.& Goetzmann,W.,2011, *Modern Portfolio Theory and Investment Analysis*. Croyden: John Wiley & Sons Ltd.

Eom, Y. H., Helwege, J., and Huang, J. 2004. Structural Models of Corporate Bond Pricing: An Emperical Analysis. *Review of Financial Studies*, 17, 499-544.

Ernst & Young., 2014, *Applying IFRS: IFRS 13 Fair Value Measurement: Credit valuation adjustments for derivative contracts*. [Online]. Available: http://www.ey.com/Publication/vwLUAssets/EY-credit-valuation-adjustments-for-derivative-contracts/$FILE/EY-Applying-FV-April-2014.pdf [2015, April 03].

Fons, J.S. 1994. Using Default Rates to Model the Term Structure of Credit Risk. *Financial Analysts Journal*, 50(5), September-October:25-32.

Friedman, M., 1952, *The Methodology of Positive Economics, Essay in Essays on Positive Economics*. University of Chicago Press.

Gregory, J., 2012a, *Counterparty Credit Risk and Credit Value Adjustment: A Continuing Challenge for Global Financial Markets.* Croydon: John Wiley & Sons Ltd.

Hayne, E. L. *Predictions of Default Probabilities in Structural Models of Debt.* Working Paper. Berkeley, California: Haas School of Business, University of California.

Holman, G., Van Breda, R., Correia, C., 2011,The use of the Merton model to quantify the default probabilities of the top 42 non-financial South African firms. *The African Finance Journal,* 13(Conference Issue), pp.1-33.

Huang, J.H., Huang, M. 2003. How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?. Working Paper. Stanford, California: Graduate School of Business, Stanford University.

Hull, J.C., 2012, *Options, Futures, and other Derivatives*. Upper Saddle River, New Jersey: Pearson Education.

Hull, J., Predescu, M., White, A. 2004. The relationship between credit default swap spreads, bond yields, and credit rating announcements. *Journal of Banking & Finance*, 28, August: 2789-2811.

Jarrow, R.A., Lando, D., Turnbull, S.M. 1997. A Markov Model for the Term Structure of Credit Risk Spreads. *The Review of Financial Studies*, 10(2), Summer:484-523.

Langohr, H.M., Langohr, P.T., 2008, *The Rating Agencies and Their Credit Ratings: What They Are, How They Work and Why They Are Relevant*. Chichester, West Sussex: John Wiley & Sons Ltd.

Leland, H., and Toft, K. 1996. Optimal capital structure, endogenous bankruptcy and term structure of credit spreads. *Journal of Finance*, 51, 987-1019.

Liang, J., Zhou, P., Zhou, Zhou, Y., & Ma, J. 2010. Valuation of Credit Default Swap with counterparty Default Risk by Structural Model. Working paper. Shanghai, China: Department of Applied Mathematics, Shanghai University of Finance and Economics.

Longstaff, F., and Schwartz, E. 1995. Valuing risky debt: A new approach. *Journal of Finance*, 50, 789-820.

Malz, M. A., 2011, *Financial Risk Management: Models, History and Institutions*. Hoboken, New Jersey: John Wiley & Sons Ltd.

Merton, R.C., (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance,* 29*,* May:449-470.

Moody's Investors Service. 2014. Rating Symbols and Definitions. Moody's Coorporation.

Numerix., 2012, *A Practical Guide to Fair Value and Regulatory CVA.* [Online]. Available: http://www.prmia.org/sites/default/files/references/SokolPresentation.pdf [2015, April 03].

R Development Core Team. 2015. R. Edition 3.2.2 [Online]. Available: http://www.r-project.org/ [2015, April 1].

Trujllo, A. & Martin, J. (2005) Structural Models and Default Probability: Application the Spanish Stock Market. *Investment Management and Financial Innovations,* 2 (1),pp.18-25.

Shimko, D., Tejima, N., and van Deventer, D. 1993. The pricing of risky debt when interest rates are stochastic. *The Journal of Fixed Income*, 3, 94-100.

Smit, L., Swart, B., Van Niekerk, F. 2003. Credit risk models in the South African context. *Investment Analysts Journal*, 57, 41-46.

Sundaram, R., and Das, S., 2010, *Derivatives: Principals and Practice*. New York: McGraw-Hill Irwin.

Tudela, M., and Young, G. 2003. A Merton-model approach to assessing the default risk of UK public companies. Working Paper, Threadneedle Street, London: Bank of England.

Vassalou, M., & Xing, Y. 2004. Default Risk in Equity Returns. *Journal of Finance*, 59, 831-868.

Venter, J. H., and Styger, P. 2008. Stuctural Default Models Applied To South African Banks. *Journal of Econometrics*, 32(1).

Wang, H. 2006. Can Stuctural Models Price Default Risk? Evidence From Bond and Credit Derivative Markets. Working Paper. Mcgill University.

Wang, W., Suo, W. 2006. Assessing Default Probabilities from Structural Credit Risk Models. Working Paper. Kingston, Ontario: Queen's School of Business, Queen's University.

Zhou, C. 2001. The Term Structure of Credit Spreads with Jump Risk. *Journal of Banking & Finance*, 25, 2015-2040.

APPENDIX A:
R Code

A1 DATA INPUT AND ESTIMATION

```
> # we begin by importing the excel folders with the necessary data
> # financial statement inputs for the companies over the 5 year period

> data.inputs.2009 <- read.table(file ="clipboard",header = TRUE)
> data.inputs.2010 <- read.table(file ="clipboard",header = TRUE)
> data.inputs.2011 <- read.table(file ="clipboard",header = TRUE)
> data.inputs.2012 <- read.table(file ="clipboard",header = TRUE)
> data.inputs.2013 <- read.table(file ="clipboard",header = TRUE)
> data.inputs.2014 <- read.table(file ="clipboard",header = TRUE)


> daily.returns.2009 <- read.table(file="clipboard",header=”T”)
> daily.returns.2010 <- read.table(file="clipboard",header=”T”)
> daily.returns.2011 <- read.table(file="clipboard",header=”T”)
> daily.returns.2012 <- read.table(file="clipboard",header=”T”)
> daily.returns.2014 <- read.table(file="clipboard",header=”T”)
```

A1.2 Risk-free rate

```
> # we set the risk free rate to be used through the initial estimation and
calculation procedure.
# riskfreerate estimated as one year prime rate as financed at Reserve bank

> riskfree.rate.2009 <- 0.15
> riskfree.rate.2010 <- 0.105
> riskfree.rate.2011 <- 0.09
> riskfree.rate.2012 <- 0.09
> riskfree.rate.2013 <- 0.085
> riskfree.rate.2014 <- 0.085
```

A1.3 Expected growth rate of equity

```
# we now require to estimate the required rate of return for each of the giv-
en companies using CAPM.
> # we do this through the use of the function return.estimate which provides
expected rate of return on company for a given risk free rate r.

> return.estimate
function (mat,r,year)
# we write a function to estimate the return on each stock using CAPM
{if( year == 2009){
       mat <- daily.returns.2009[,2:ncol(daily.returns.2009)]}
if( year == 2010){
       mat <- daily.returns.2010[,2:ncol(daily.returns.2010)]}
if( year == 2011){
       mat <- daily.returns.2011[,2:ncol(daily.returns.2011)]}
if( year == 2012){
       mat <- daily.returns.2012[,2:ncol(daily.returns.2012)]}
if( year == 2013){
       mat <- daily.returns.2013[,2:ncol(daily.returns.2013)]}
if( year == 2014){
```

```
        mat <- daily.returns.2014[,2:ncol(daily.returns.2014)]}

r <- r
covar <- cov(mat)

mew <- matrix(NA,nrow=nrow(covar),ncol=3)
Rbar <- mean(mat[,1])*252
name <- rownames(covar)
rownames(mew) <- name
colnames(mew) <- c("B","Rm-Rf","U")

for (i in 1:nrow(covar))
{
        mew[i,1] <- covar[i,1]/covar[1,1]
        mew[i,2] <- Rbar - r
        mew[i,3] <- r+mew[i,1]*mew[i,2]
}
mu <- matrix(mew[2:ncol(covar),3])
colnames(mu) <- c("U")
rownames(mu) <- rownames(covar[2:nrow(covar),])
return(mu)

}
```

A1.4 Volatility of equity

```
> # we now estimate the volatility of the returns of shares for each respec-
tive company using a garch(1,1) model
> library(fGarch)
> library(timeSeries)
> # we estimate volatility of equities through garch(1,1) in the following
function

# creating return structure without the market returns
> returns.2009 <- daily.returns.2009[,-2]
> returns.2010 <- daily.returns.2010[,-2]
> returns.2011 <- daily.returns.2011[,-2]
> returns.2012 <- daily.returns.2012[,-2]
> returns.2013 <- daily.returns.2013[,-2]
> returns.2014 <- daily.returns.2014[,-2]



> vol.estimate
function (mat,year)
{
if( year == 2009)
{
        mat <- as.timeSeries(returns.2009)
}
 if( year == 2010)
{
        mat <- as.timeSeries(returns.2010)
}
 if( year == 2011)
{
        mat <- as.timeSeries(returns.2011)
}
 if( year == 2012)
{
```

```
       mat <- as.timeSeries(returns.2012)
}
 if( year == 2013)
{
       mat <- as.timeSeries(returns.2013)
}
 if( year == 2014)
{
       mat <- as.timeSeries(returns.2014)
}

 coeff.est <- matrix(NA,nrow=ncol(mat),ncol=1)
 name <- colnames(mat)
 rownames(coeff.est) <- name
 colnames(coeff.est) <- c("Sigma.E")


 for (i in 1:ncol(mat))
 {
       fit <- garchFit(formula= ~ garch(1,1) ,data=mat[,i],cond.dist="std")
       coeff.est[i,1] <- mean(fit@sigma.t)*sqrt(252)
}
 equity.volatility <- matrix(coeff.est)
 colnames(equity.volatility) <- c("equitty.vol")
 rownames(equity.volatility) <- colnames(mat)
 return(equity.volatility)

 }

> # we now combine the estimates of equity volatility along with the debt
structrues of the companies as recorded in Company.Structure.

> data.inputs.2009 <-
cbind(vol.estimate(,2009),data.inputs.2009[,2:ncol(data.inputs.2009)])
> data.inputs.2010 <-
cbind(vol.estimate(,2010),data.inputs.2010[,2:ncol(data.inputs.2010)])
> data.inputs.2011 <-
cbind(vol.estimate(,2011),data.inputs.2011[,2:ncol(data.inputs.2011)])
> data.inputs.2012 <-
cbind(vol.estimate(,2012),data.inputs.2012[,2:ncol(data.inputs.2012)])
> data.inputs.2013 <-
cbind(vol.estimate(,2013),data.inputs.2013[,2:ncol(data.inputs.2013)]).
> data.inputs.2014 <-
cbind(vol.estimate(,2014),data.inputs.2014[,2:ncol(data.inputs.2014)])
```

A2 MERTON MODEL

```
> # we now create the variables required in order to solve simultaneous equa-
tions for sigma and vt. the volatolity of asset value and the asset value of
the firm
> merton.solve
function (mat,year,T,D.mult=1,Vol.mult=1)
{
# function that solves for the merton model probability of default under risk
free rate r specified in the default
if( year == 2009)
{
mat <- data.inputs.2009
r <- riskfree.rate.2009
```

```
}
if( year == 2010)
{
mat <- data.inputs.2010
r <- riskfree.rate.2010
}
if( year == 2011)
{
mat <- data.inputs.2011
r <- riskfree.rate.2011
}
if( year == 2012)
{
mat <- data.inputs.2012
r <- riskfree.rate.2012
}
if( year == 2013)
{
mat <- data.inputs.2013
r <- riskfree.rate.2013
}
if( year == 2014)
{
mat <- data.inputs.2014
r <- riskfree.rate.2014
}


# create an output matrix for the paramaters we are estimating as output of
convergence
mer.est1 <- matrix(NA,nrow=nrow(mat),ncol=6)
rownames(mer.est1) <- rownames(mat)
colnames(mer.est1) <- c("Vt","SigmaV","conv","L","RN Pd","RW Pd")

# now estimate expected return on each company for given risk free interest
rate
Mu <- return.estimate(,r,year)


# for loop to run through each company
for(i in 1:nrow(mat))
{
# creating simulatenous equations to be solved under merton model
simultaneous.equations.merton <- function (x)
       {
              R <- r # set interest rate to specified one
              T1 <- T # merton implicitly calculates over 1 year period
              sigmaS <- mat[i,1]*Vol.mult # input company[i] equity volatili-
ty
              S0 <- mat[i,2] # input comp[i] equity value
              D <- mat[i,3]*D.mult # input comp[i] debt value multiplied by
the stress factor

              y <- numeric(2)

              L  <- exp(-R*T1)*D/x[1]
              d1 <- (log(1/L)+(0.5*(x[2]^2))*T1)/(x[2]*sqrt(T1))
              d2 <- d1 -x[2]*sqrt(T1)

              # simultaneous equations to be solved that yield Vt and SigmaT
              y[1] <- S0-(x[1]*pnorm(d1)-D*exp(-R*T1)*pnorm(d2))
```

```
               y[2] <- sigmaS*(x[1]*pnorm(d1)-D*exp(-R*T1)*pnorm(d2))-
x[2]*x[1]*pnorm(d1)
               y


        }
# input starting values by using excel answers as input due to sensitivity of
starting values and neg vol induction
xstart <- c(mat[i,2]+mat[i,3],mat[i,1]*(mat[i,2]/(mat[i,2]+mat[i,3])))
# create a solutions output vector
solutions <-
nleqslv::nleqslv(xstart,simultaneous.equations.merton,method="Newton")
# fill matrix with estimates required as well as indication of convergence to
solution in newton method
mer.est1[i,1] <- solutions$x[1] # Vt estimate
mer.est1[i,2] <- solutions$x[2] # Sigma est.
mer.est1[i,3] <- abs(solutions$fvec[1])+ abs(solutions$fvec[2]) # convergence
reached in sol.

# naming of output variables
D <- mat[i,3]*D.mult
Vt <- mer.est1[i,1]
Sigma <- mer.est1[i,2]

# creating output values for Leverage, Risk neutral pd and real world pd
mer.est1[i,4] <- exp(-r*T)*D/Vt # leverage
mer.est1[i,5] <- pnorm((log(D/Vt)-(r-0.5*((Sigma)^2))*T)/(Sigma*sqrt(T))) #
risk-neutral pd
mer.est1[i,6] <- pnorm((log(D/Vt)-(Mu[i]-0.5*((Sigma)^2))*T)/(Sigma*sqrt(T)))
# real world pd

}


bloom.pd <- matrix(mat[,5],nrow=nrow(mat),ncol=1)
colnames(bloom.pd) <- c("BloomPD")

results <- cbind(Mu,mer.est1,bloom.pd)

return(results)


}
```

A3 DELIANEDIS-GESKE MODEL

```
> library("pbivnorm", lib.loc="~/R/win-library/3.2")


> DG.solve
function (mat,year,T1,T2,D.mult=1,Vol.mult=1)
{
# function that solves for the merton model probability of default under risk
free rate r specified in the default
p <- sqrt(T1/T2)


if( year == 2009)
{
mat <- data.inputs.2009
r <- riskfree.rate.2009
```

```
}
if( year == 2010)
{
mat <- data.inputs.2010
r <- riskfree.rate.2010
}
if( year == 2011)
{
mat <- data.inputs.2011
r <- riskfree.rate.2011
}
if( year == 2012)
{
mat <- data.inputs.2012
r <- riskfree.rate.2012
}
if( year == 2013)
{
mat <- data.inputs.2013
r <- riskfree.rate.2013
}
if( year == 2014)
{
mat <- data.inputs.2014
r <- riskfree.rate.2014
}

# create an output matrix for the paramaters we are estimating as output of
convergence
DG.est <- matrix(NA,nrow=nrow(mat),ncol=10)
rownames(DG.est) <- rownames(mat)
colnames(DG.est) <-
c("Vt","V*","Sigma","Conv","RN.S","RN.Total","RN.LT","RW.S","RW.Total","RW.LT
")

# now estimate expected return on each company for given risk free interest
rate
Mu <- return.estimate(,r,year)


# for loop to run through each company
for(i in 1:nrow(mat))
{
# creating simulatenous equations to be solved under Delianedis and Geske
model
simultaneous.equations.DG <- function (x)
        {
                R <- r # set interest rate to specified one
                T1 <- T1 # short term debt over 1 year
                sigmaS <- mat[i,1]*Vol.mult # input company[i] equity volatili-
ty
                S0 <- mat[i,2] # input comp[i] equity value
                D1 <- mat[i,3]*D.mult # input comp[i] short term debt value
                D2 <- mat[i,4]*D.mult # input comp[i] long term debt value

                y <- numeric(3)

                B2t1 <- D2*exp(-R*(T2-T1))
                p <- sqrt(T1/T2)
                d1 <- (log(x[1]/x[2])+(R+0.5*((x[3])^2))*T1)/(x[3]*sqrt(T1))
                d2 <- (log(x[1]/D2)+(R+0.5*((x[3])^2))*T2)/(x[3]*sqrt(T2))
```

```
               # simultaneous equations to be solved that yield Vt and SigmaT
and V*
               y[1] <- S0 -
(x[1]*pbivnorm(d1+x[3]*sqrt(T1),d2+x[3]*sqrt(T2),p)-D2*exp(-
R*T2)*pbivnorm(d1,d2,p)-D1*exp(-R*T1)*pnorm(d1))
               y[2] <- sigmaS*S0-pnorm(d1)*x[1]*x[3]
               y[3] <- x[2]- (D1 + B2t1)

               y
       }
# input starting values
xstart <- c(mat[i,2]+mat[i,3]+mat[i,4],mat[i,2],mat[i,1])
# create a solutions output vector
solutions <-
nleqslv::nleqslv(xstart,simultaneous.equations.DG,method="Newton")
# fill matrix with estimates required as well as indication of convergence to
solution in newton method
DG.est[i,1] <- solutions$x[1] # Vt estimate
DG.est[i,2] <- solutions$x[2] # V* est.
DG.est[i,3] <- solutions$x[3] # Sigma est.
DG.est[i,4] <- abs(solutions$fvec[1])+ abs(solutions$fvec[2])+
abs(solutions$fvec[3])

# creating names of output variables
Vt <- DG.est[i,1]
Vstar <- DG.est[i,2]
Sigma <- DG.est[i,3]
D2 <- mat[i,4]*D.mult # input comp[i] long term debt value
rn.d1 <- (log(Vt/Vstar)+(r+0.5*((Sigma)^2))*T1)/(Sigma*sqrt(T1))
rn.d2 <- (log(Vt/D2)+(r+0.5*((Sigma)^2))*T2)/(Sigma*sqrt(T2))
rw.d1 <- (log(Vt/Vstar)+(Mu[i]+0.5*((Sigma)^2))*T1)/(Sigma*sqrt(T1))
rw.d2 <- (log(Vt/D2)+(Mu[i]+0.5*((Sigma)^2))*T2)/(Sigma*sqrt(T2))

# risk neutral and real world probabilities
DG.est[i,5] <- 1-pnorm(rn.d1)
DG.est[i,6] <- 1-pbivnorm(rn.d1,rn.d2,p)
DG.est[i,7] <- 1-pbivnorm(rn.d1,rn.d2,p)/pnorm(rn.d1)


DG.est[i,8] <- 1-pnorm(rn.d1)
DG.est[i,9] <- 1-pbivnorm(rw.d1,rn.d2,p)
DG.est[i,10] <- 1-pbivnorm(rw.d1,rn.d2,p)/pnorm(rn.d1)

}

bloom.pd <- matrix(mat[,5],nrow=nrow(mat),ncol=1)
colnames(bloom.pd) <- c("BloomPD")
results <- cbind(Mu,mer.est1,bloom.pd)

return(results)


}
```

A4 GRAPHICS

A4.1 Merton 3D-graphics and level curves

```
> merton.graph
#function to create 3-D graphic of Merton PD as function of leverage and vol-
atility
function (start=0,end=1,len=70,theta=0,phi=30)
{
pts <-seq(from=start,to=end,len=len)
fun1 <-function(x,y)
{
 pnorm(-(log(1/x)+(0.5*(y^2))*1)/(y*sqrt(1))+y*sqrt(1))
 }
zz <- outer(pts,pts,fun1)
persp(x=pts,y=pts,z=zz,theta=theta,phi=phi,ticktype="detailed",xlab="Leverage
",ylab="Firm Volatility")
}

> merton.contour
#function to create graphic of contour levels of Merton PD for leverage and
volatility combinations.
function (level=0.00001)
{
f <-function(x,y)
{
 pnorm(-(log(1/x)+(0.5*(y^2))*1)/(y*sqrt(1))+y*sqrt(1))
 }
 x <- seq(0,1,length=100)
 y <- x
 z <- outer(x,y,f)
 con-
tour(x=x,y=x,z=z,levels=level,drawlabels=FALSE,lwd=3,xlab="Leverage",ylab="Fi
rm Value Volatility")

 }
```

A4.2 D&G 3D-graphics and level curves

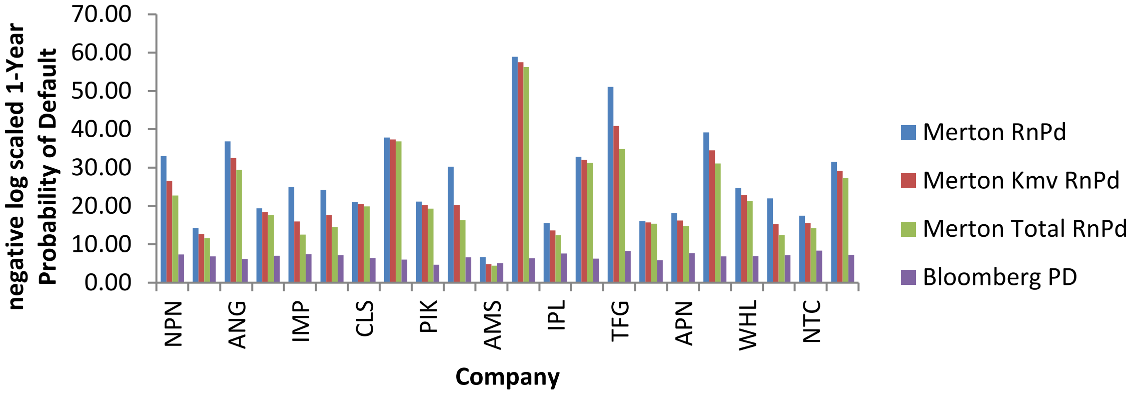
```
> DG.graphic.total
#function to create 3-D graphic of D&G total PD as function of leverage and
volatility for given short run leverage

function (start=0.00001,end=1,len=70,theta=0,phi=30)
{
pts <-seq(from=start,to=end,len=len)
require(pbivnorm)
{
p <- sqrt(1/5)
d1 <-  ((log(1/0.2)+(0.5*(y^2))*1)/(y*sqrt(1))+y*sqrt(1))
d2 <- (-(log(1/x)+(0.5*(y^2))*4)/(y*sqrt(4))+y*sqrt(4))
pbivnorm(d1,d2,p)
 }
zz <- outer(pts,pts,fun1)
persp(x=pts,y=pts,z=zz,theta=theta,phi=phi,ticktype="detailed",xlab=" long-
term leverage",ylab="Firm Volatility")


}
```

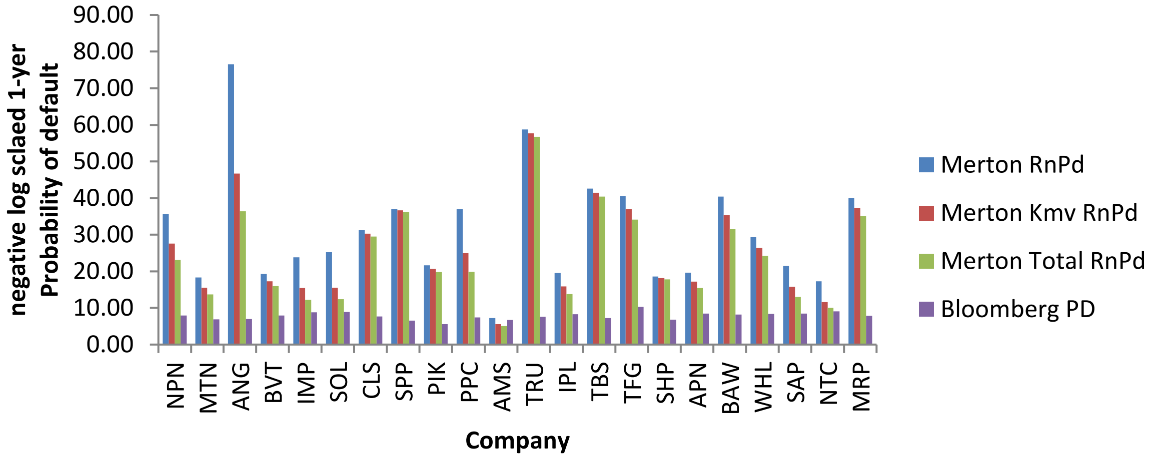
```
> DG.contour
#function to create graphic of contour levels of D&G total PD for leverage
and volatility combinations of a given short run leverage.

function (level=0.0001)
{
f <-function(x,y)
{

p <- sqrt(1/5)
d1 <-  ((log(1/0.2)+(0.5*(y^2))*1)/(y*sqrt(1))+y*sqrt(1))
d2 <- (-(log(1/x)+(0.5*(y^2))*4)/(y*sqrt(4))+y*sqrt(4))
pbivnorm(d1,d2,p)
 }
 x <- seq(0.000001,1,length=100)
 y <- x
 z <- outer(x,y,f)
 con-
tour(x=x,y=x,z=z,levels=level,drawlabels=FALSE,lwd=3,xlab="Leverage",ylab="Fi
rm Value Volatility")

 }
```

APPENDIX B:
Merton Model Results

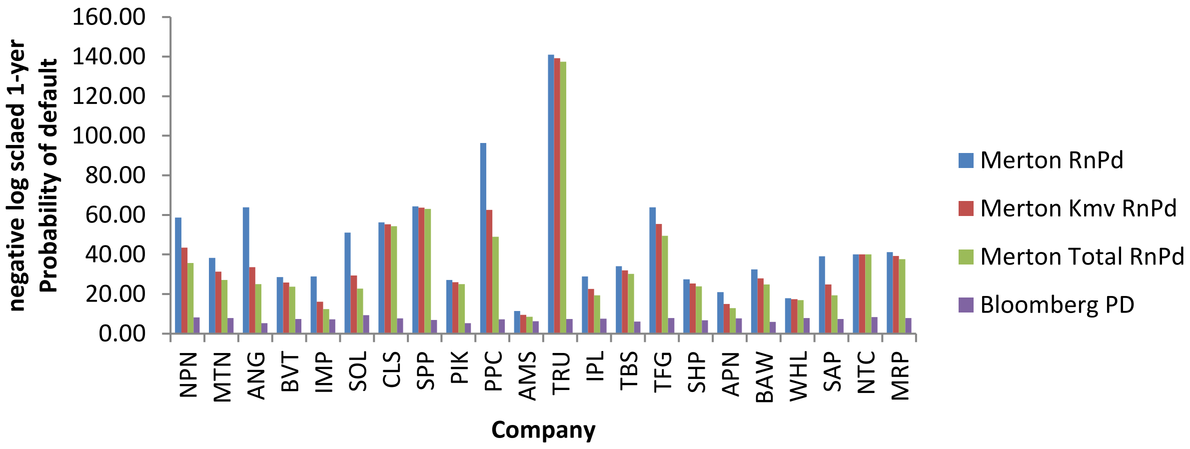
B1 MERTON RESULTS ONE-YEAR DEFAULT PROBABILITIES

Figure B.1 Merton model(s) one-year default probabilities for 2009-2014






120.00
100.00
80.00
60.00
40.00
20.00
0.00
negative log scaled 1-yer Probability of default
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
AMS
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
Merton RnPd
Merton Kmv RnPd
Merton Total RnPd
Bloomberg PD


160.00
140.00
120.00
100.00
80.00
60.00
40.00
20.00
0.00
negative log sclaed 1-yer Probability of default
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
AMS
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
Merton RnPd
Merton Kmv RnPd
Merton Total RnPd
Bloomberg PD

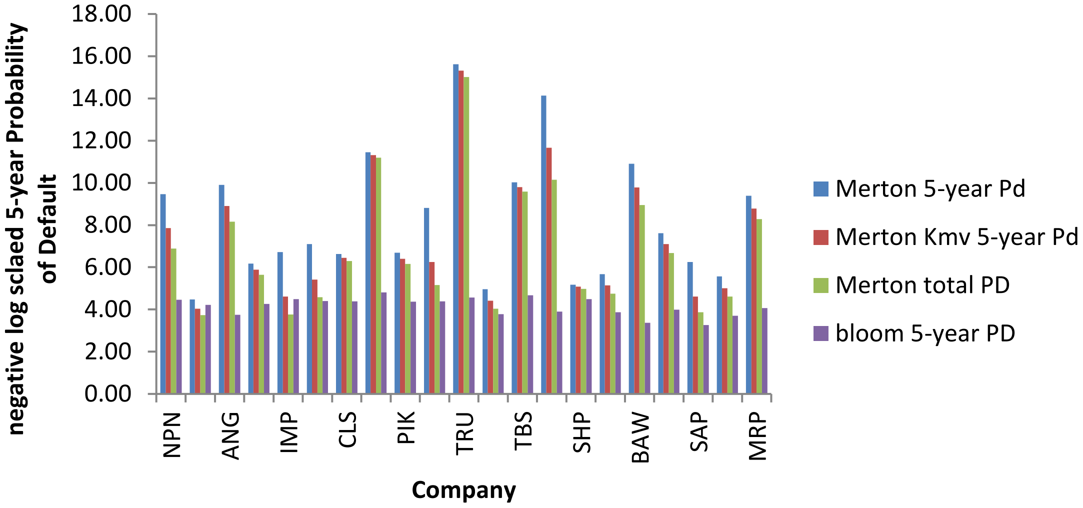

200.00
150.00
100.00
50.00
0.00
negative log sclaed 1-yer Probability of default
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
AMS
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
Merton RnPd
Merton Kmv RnPd
Merton Total RnPd
Bloomberg PD

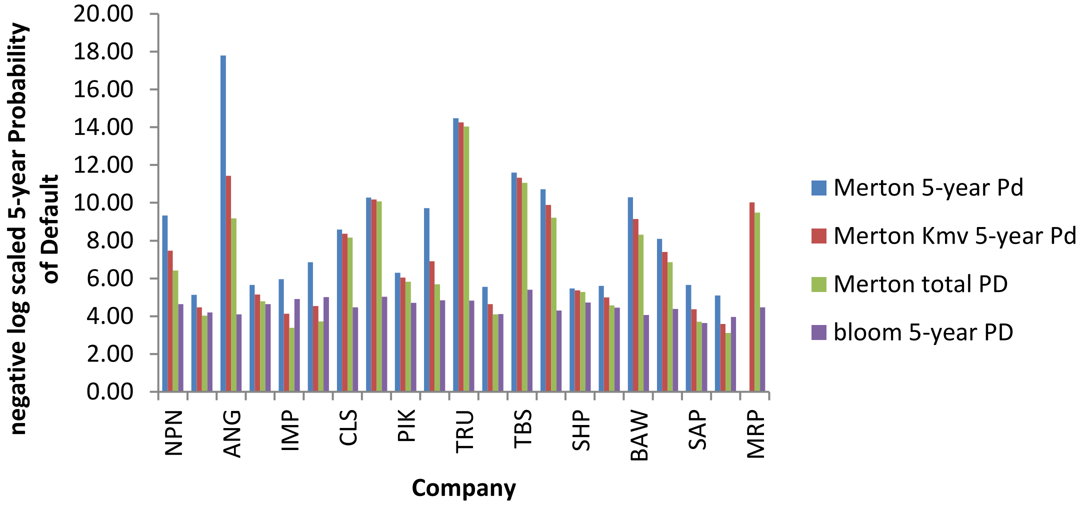
Table B.1: AMS results Merton models

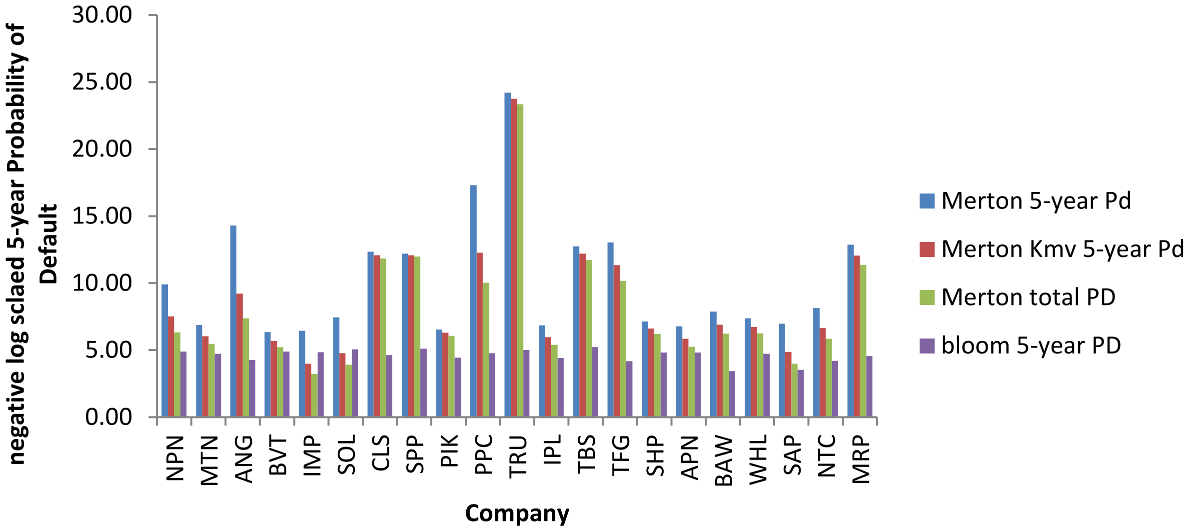
	2009	2010	2011	2012	2013	2014
equit-ty,vol	0,4796403	0,4830579	0,4484845	0,4207861	0,3486897	0,3178090
ET	46,0000000	64,5500000	80,5000000	87,8000000	98,8900000	97,8600000
D1	36,7479918	39,0929091	65,4325426	65,706543	73,7206648	75,20358623
D2	153,1386162	85,8056593	66,8315438	89,467214	94,9795064	95,5968257
BloomPD	0,006239271	0,0012333	0,00120251	0,0031786	0,00189367	0,001584752
Merton L	0,407456051	0,3528656	0,42624157	0,4061624	0,40643453	0,413785286
Merton RnPd	0,001284419	0,0007493	0,00072587	0,0002503	1,0854E-05	1,71506E-06
Merton RwPd	0,00120635	0,0007504	0,00071725	0,000251	1,194E-05	1,73985E-06
Merton kmv L	0,679690213	0,5335773	0,52883008	0,5348111	0,52959612	0,535854181
Merton kmv RnPd	0,007729974	0,0038042	0,00183018	0,0009757	7,4392E-05	1,62138E-05
Merton kmv RwPd	0,007018288	0,0038110	0,00180574	0,0009786	8,2922E-05	1,64814E-05
Merton Total L	0,780608697	0,6354562	0,60031386	0,6176568	0,61043098	0,615847518
Merton Total RnPd	0,011391182	0,0066731	0,00296551	0,001803	0,00018172	4,62608E-05
Merton Total RwPd	0,009961104	0,0066873	0,00292099	0,0018093	0,0002057	4,71349E-05
M Sigma V	0,284364386	0,3126896	0,25740529	0,249905	0,20697111	0,186304423
M kmv Sigma V	0,154528507	0,2258017	0,21153045	0,1958533	0,16403172	0,147511027
M Tot Sigma V	0,106291383	0,1769168	0,17959746	0,161077	0,13585455	0,122090731

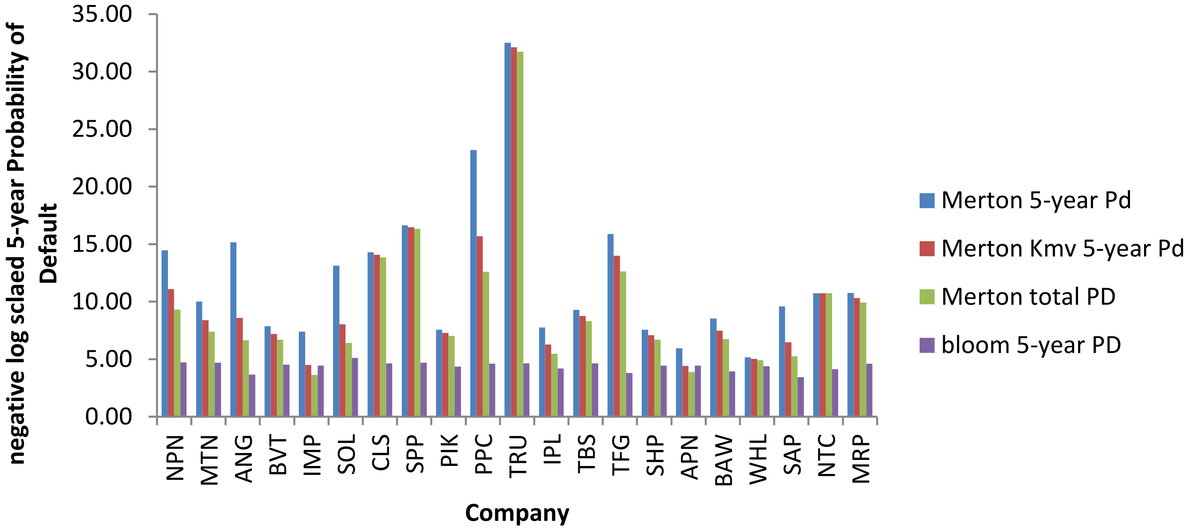
B2 MERTON RESULTS FIVE-YEAR DEFAULT PROBABILITIES

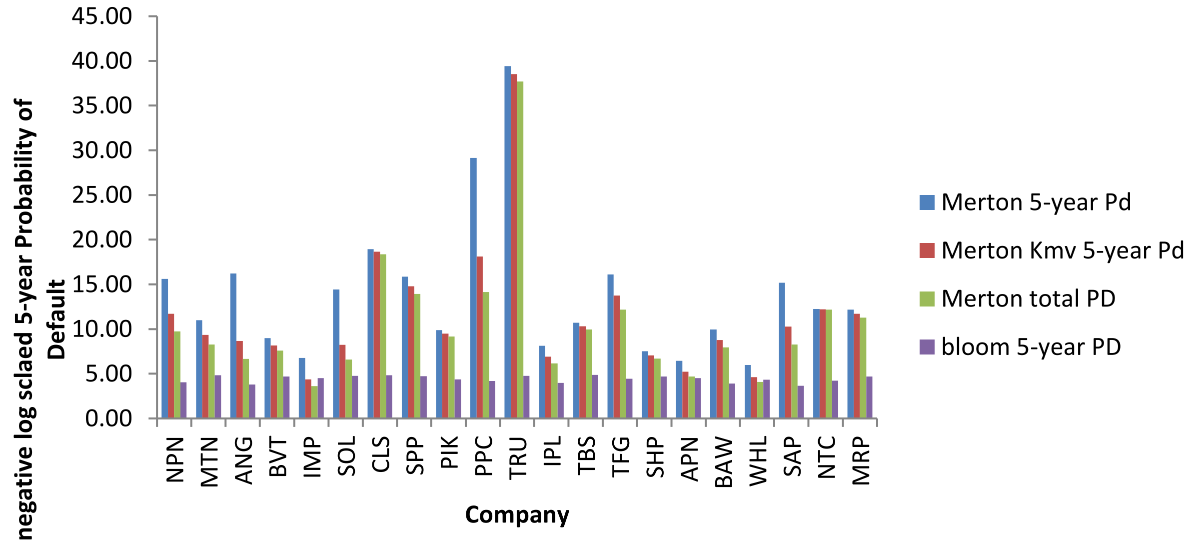
Figure B.2 Merton model(s) five-year default probabilities for 2009-2014






negative log sclaed 5-year Probability of Default
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
Merton 5-year Pd
Merton Kmv 5-year Pd
Merton total PD
bloom 5-year PD


negative log sclaed 5-year Probability of Default
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
Merton 5-year Pd
Merton Kmv 5-year Pd
Merton total PD
bloom 5-year PD


negative log sclaed 5-year Probability of Default
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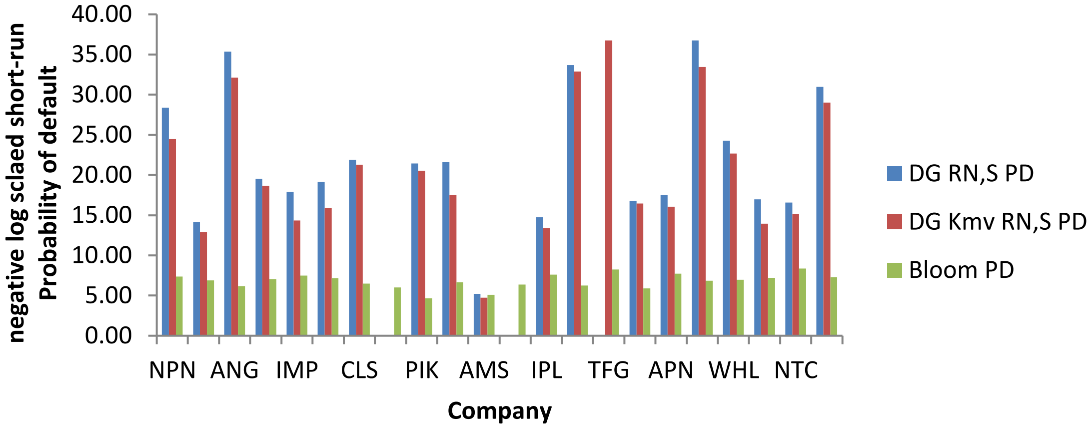
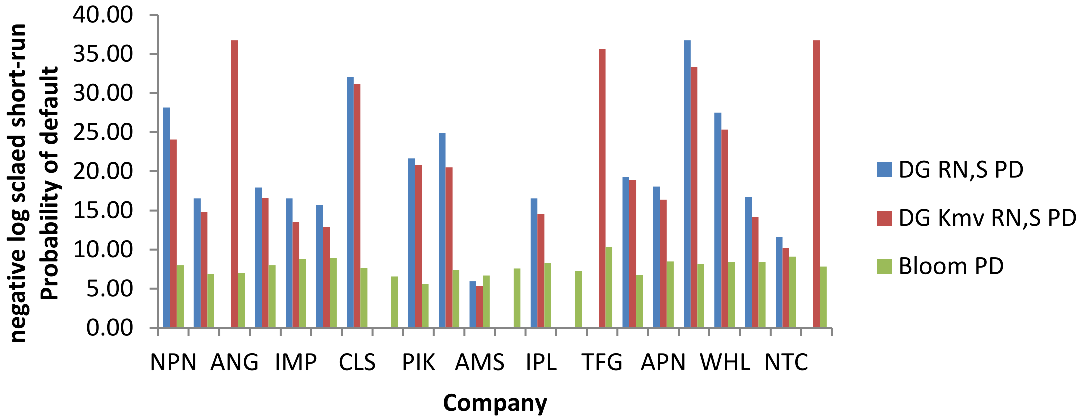
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
Merton 5-year Pd
Merton Kmv 5-year Pd
Merton total PD
bloom 5-year PD

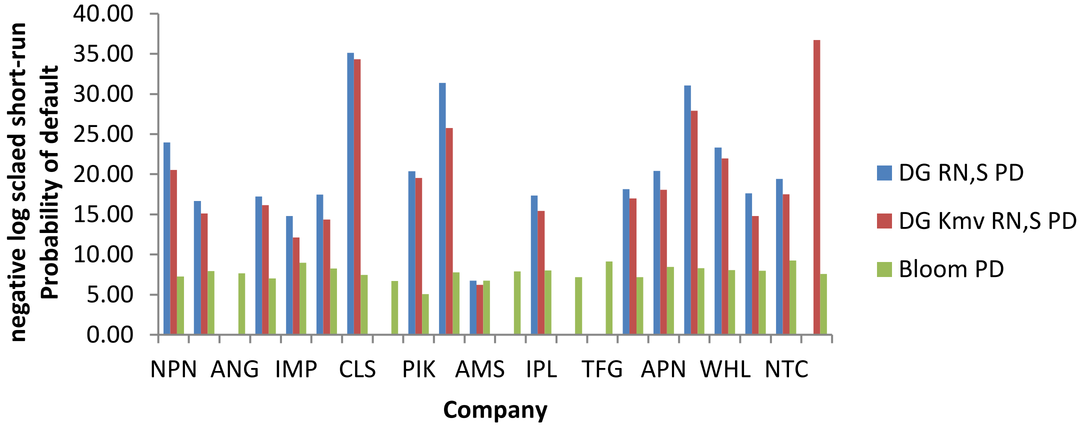
APPENDIX C:
Delianedis & Geske Results

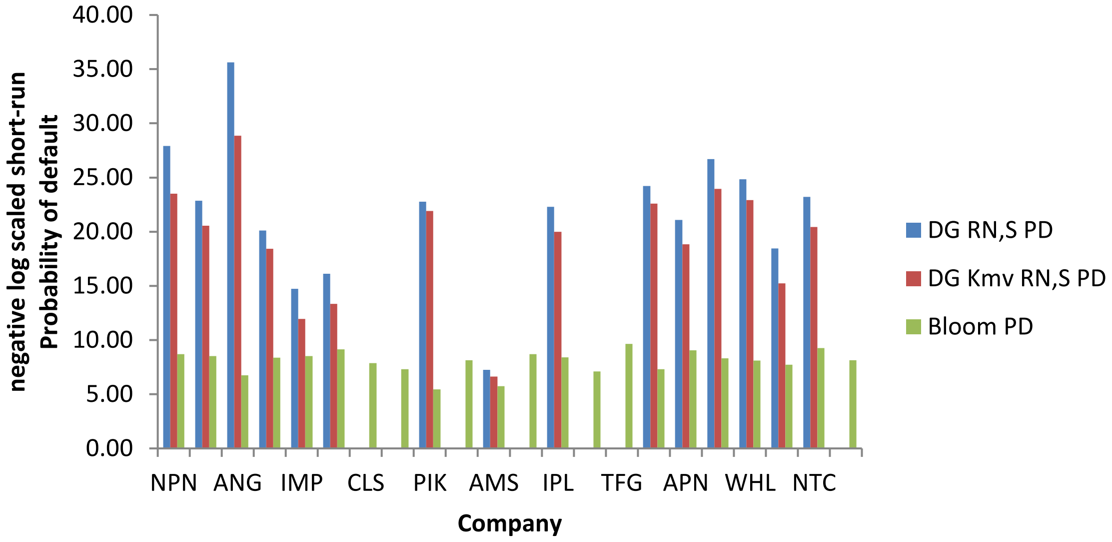
C1 D&G RESULTS ONE-YEAR DEFAULT PROBABILITIES

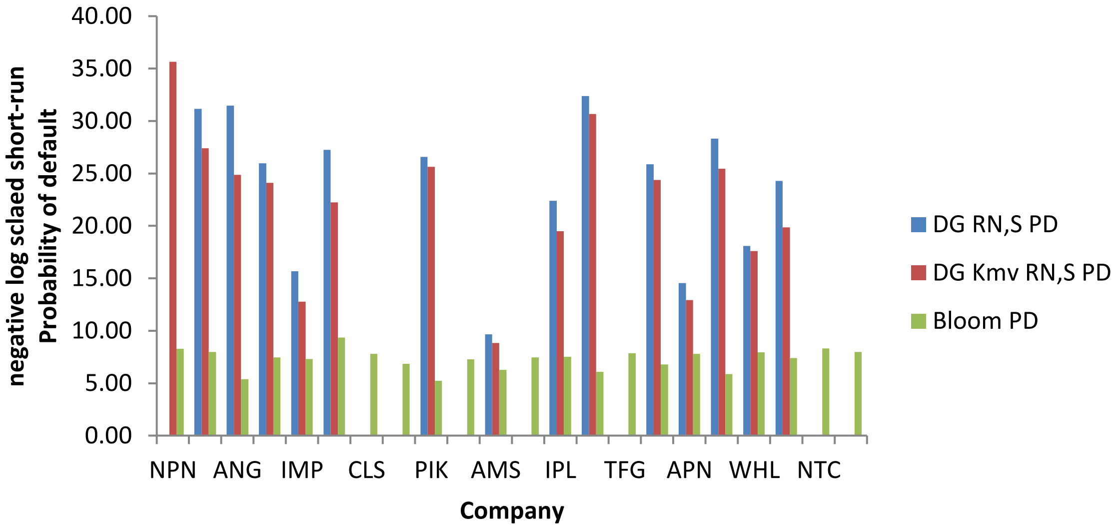
Figure C.1 D&G model(s) short run default probabilities for 2009-2014








negative log scaled short-run Probability of default
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN ANG IMP CLS PIK AMS IPL TFG APN WHL NTC
Company
DG RN,S PD
DG Kmv RN,S PD
Bloom PD


negative log sclaed short-run Probability of default
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN ANG IMP CLS PIK AMS IPL TFG APN WHL NTC
Company
DG RN,S PD
DG Kmv RN,S PD
Bloom PD

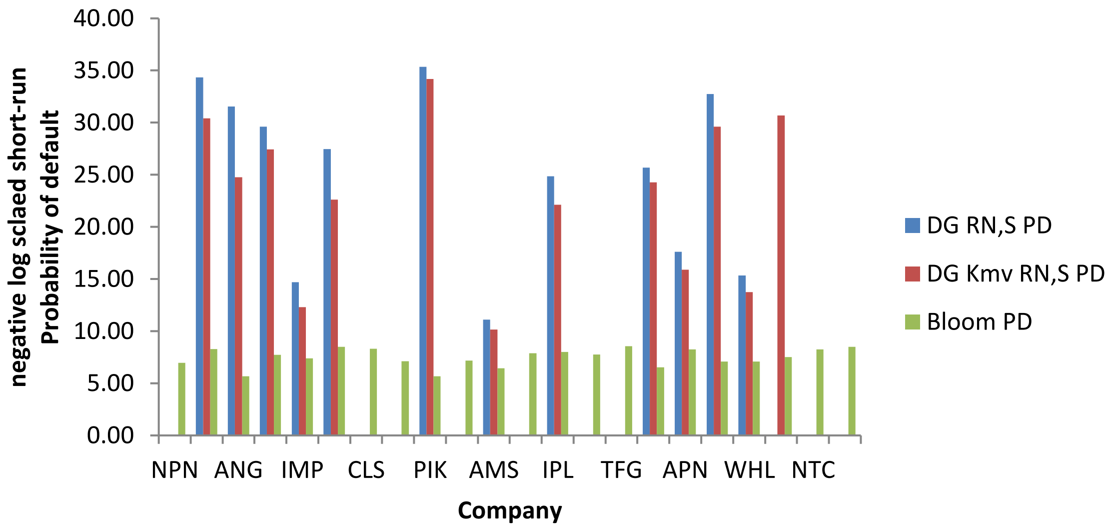
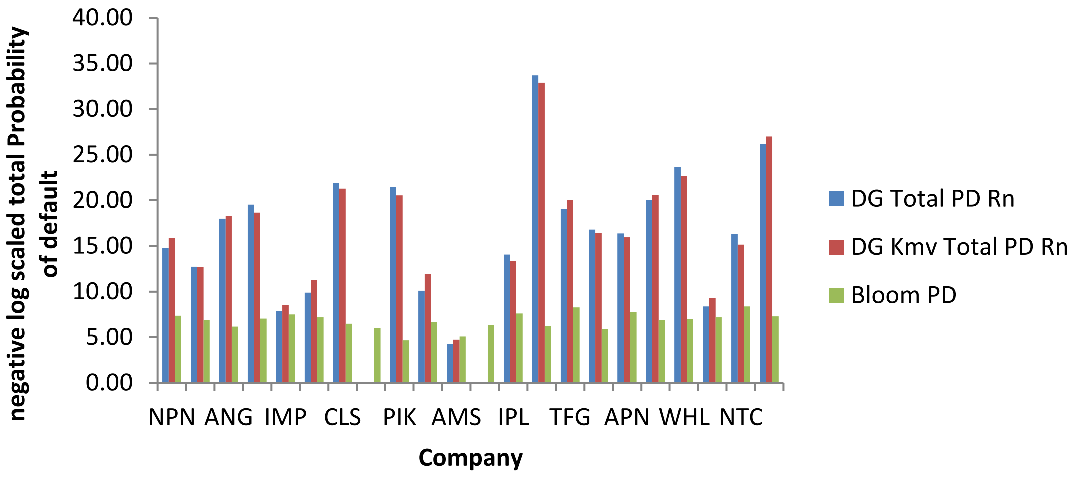

negative log sclaed short-run Probability of default
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN ANG IMP CLS PIK AMS IPL TFG APN WHL NTC
Company
DG RN,S PD
DG Kmv RN,S PD
Bloom PD

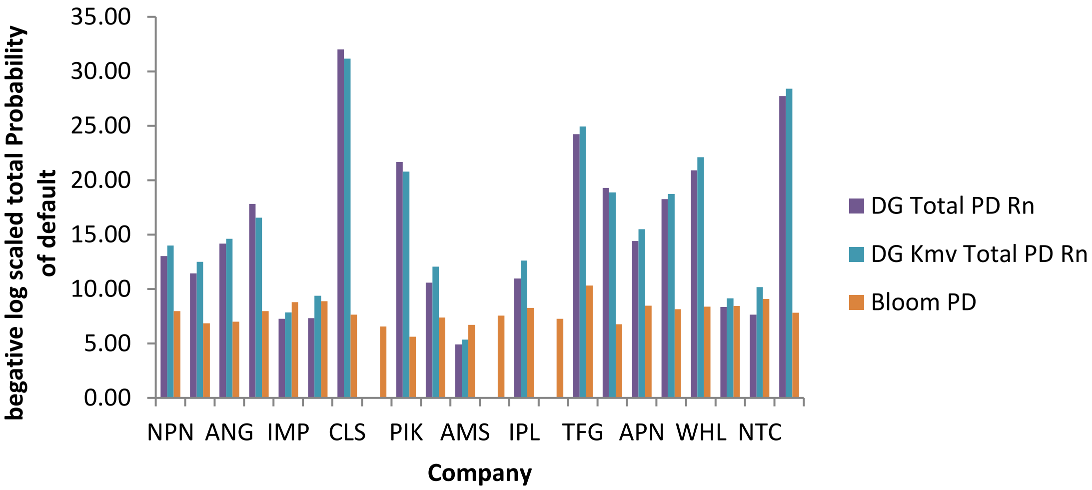
Table C.1: AMS results D&G models

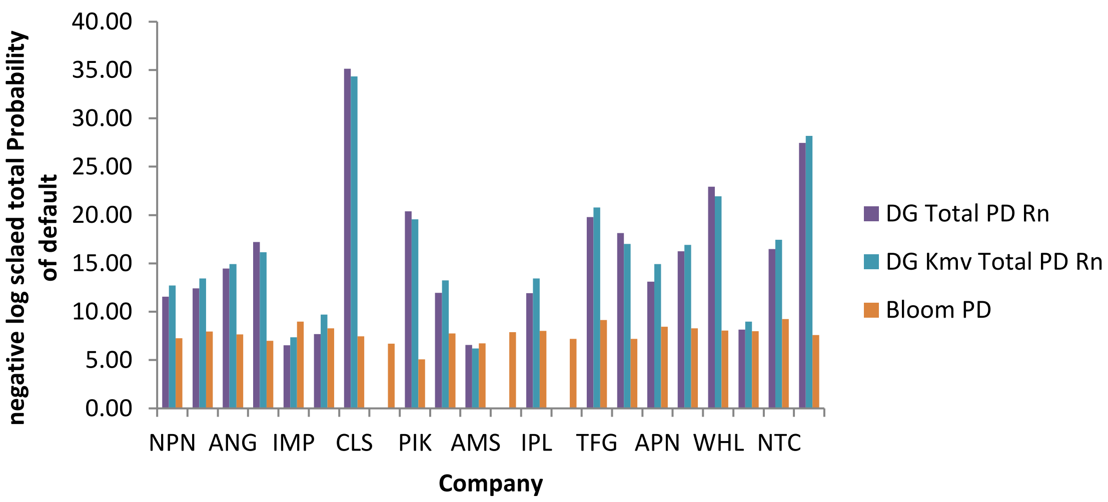
	2009	**2010**	**2011**	**2012**	**2013**	**2014**
DG Vt	149,8122004	150,4102082	182,901247	204,88694	228,697323	229,4336018
DK kmv Vt	215,8050168	189,0872239	213,4344	245,76867	272,316854	273,3369059
DG V*	120,7922463	95,47124471	112,059329	128,1257	141,324259	143,2465697
DG Kmv V*	197,3615544	138,3740744	145,4751	172,85931	188,814012	191,0449826
DG Sigma	0,148092997	0,207846669	0,19762983	0,1804472	0,15078498	0,135556687
DG kmv Sigma	0,103150755	0,165674055	0,16949257	0,1505202	0,12664249	0,113786283
DG RN,S	0,005529798	0,002586892	0,00120996	0,00071	6,372E-05	1,52484E-05
DG kmv RN,S	0,008849667	0,004643255	0,00200519	0,0013012	0,00014385	3,87696E-05
DG Short term L	0,693981195	0,571471204	0,55994432	0,5715253	0,56759767	0,573471945
DG kmv short term L	0,787148819	0,658857695	0,62292785	0,6428057	0,63686102	0,641981231
DG Long term lever-age	0,598723483	0,491219439	0,46109748	0,4829159	0,48225783	0,486694833
DG kmv long term lever-age	0,415634834	0,390742519	0,39513455	0,4025865	0,40501009	0,408522033
Bloom PD	0,006239271	0,001233279	0,00120251	0,0031786	0,00189367	0,001584752

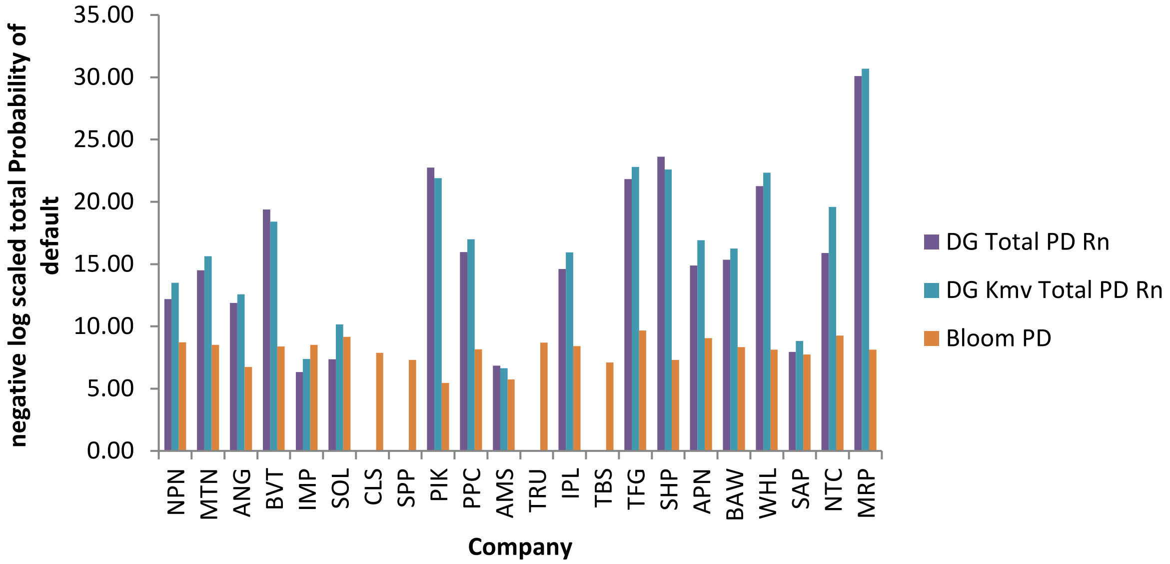
C2 D&G RESULTS TOTAL DEFAULT PROBABILITIES

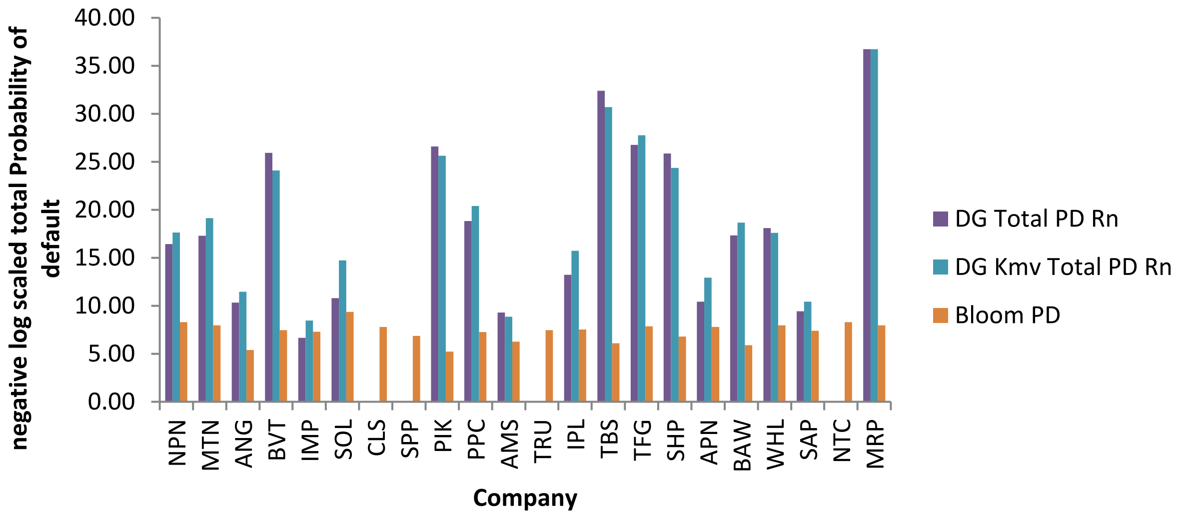
Figure C.2 D&G model(s) total default probabilities for 2009-2014








negative log scaled total Probability of default
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
AMS
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
DG Total PD Rn
DG Kmv Total PD Rn
Bloom PD


negative log scaled total Probability of default
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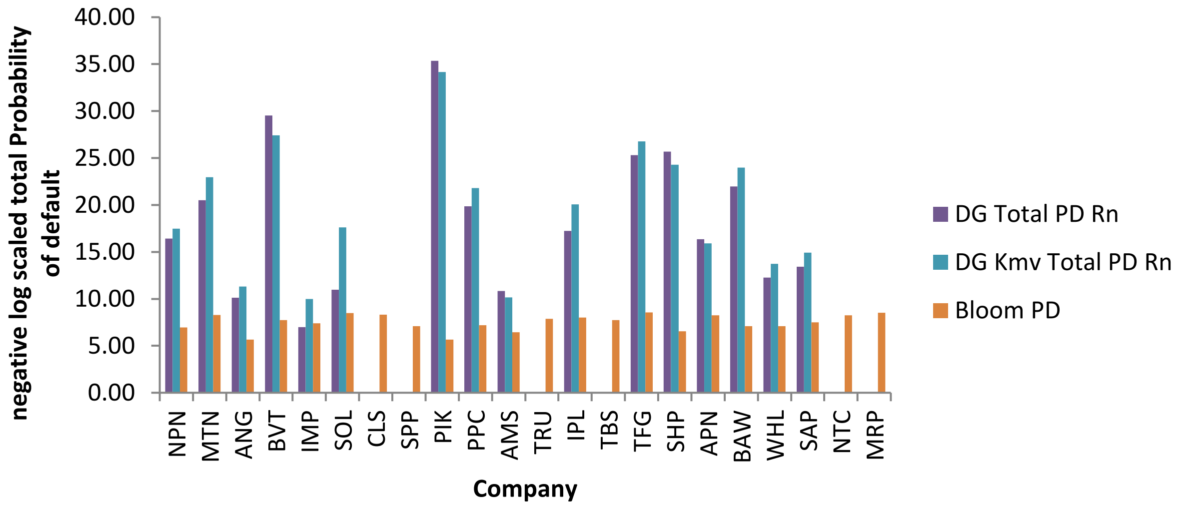
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
AMS
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
DG Total PD Rn
DG Kmv Total PD Rn
Bloom PD


negative log scaled total Probability of default
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
NPN
MTN
ANG
BVT
IMP
SOL
CLS
SPP
PIK
PPC
AMS
TRU
IPL
TBS
TFG
SHP
APN
BAW
WHL
SAP
NTC
MRP
Company
DG Total PD Rn
DG Kmv Total PD Rn
Bloom PD