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The single- and multi-curve approach to swap valuation along with zero-coupon risk free swap curve construction

by

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ABSTRACT

Determining the risk neutral value of cash collateralised derivatives is important. This is because the fair value of a cash collateralised derivative is determined by adding the fair value for non-performance risks to the risk neutral value where a risk free rate is needed for discounting (Gregory, 2012:306). Throughout the considered cash collateralised derivative is a “plain vanilla” interest rate swap, which is assumed to be fully collateralised such that no adjustments need to be made for credit risk.

The 2008 financial crisis lead institutions to consider a more appropriate risk free rate to discount cash collateralised derivatives in their fair value calculation (Schubert, 2012:28). Thus, there has been a shift in global markets to using the multi-curve approach to swap valuation where the OIS zero-coupon risk free curve is used for discounting swaps.

There are markets that have not made the transition to using the multi-curve swap valuation approach and instead still use the single-curve swap valuation approach. This could lead to a possible error in swap value calculation for these markets. Observed market swap curves are used when calculating the value of a swap. It is important to know how these swap curves are constructed and how variations in the construction of these swap curves could influence swap values.

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LIST OF ABBREVIATIONS AND DEFINITIONS

CVA	Credit valuation adjustment
DVA	Debit valuation adjustment
FRA	Forward Rate Agreement
GBP	Great Britain Pound
IFRS	International Financial Reporting Standards
JIBAR	Johannesburg Interbank Agreed Rate
LIBOR	London Interbank Offered Rate
OIS	Overnight index swap
UK	United Kingdom

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CHAPTER 1
INTRODUCTION

The International Accounting Standards Board provided a framework for fair value measurement by the issuance of IFRS 13 Fair Value Measurement in May 2011 with it being effective for financial periods starting 1 January 2013 (IASB, 2014). The intention of IFRS 13 is to reduce inconsistencies applied to fair value measurement of assets and liabilities in practise and to enhance the comparability of fair value information disclosed between entities.

The fair value of cash collateralised derivatives consists of two parts, the first is the calculation of fair value in the risk neutral world where a risk free rate is needed for discounting, and the second is where the adjustments to the fair value for non-performance risks are added (Gregory, 2012:306). The adjustments for non-performance risks are referred to as fair value adjustments of Credit and Debit Value Adjustments (CVA/DVA). The 2008 financial crisis lead institutions to consider a more appropriate risk free rate to discount cash collateralised derivatives in their fair value calculation (Schubert, 2012:28).

The cash collateralised derivative that is used throughout this paper will be the “plain vanilla” interest rate swap, which will be referred to as a swap. It will be assumed that the swap is fully collateralised, and therefore no adjustments need be made for credit risk.

In the South African market, the 3-month zero-coupon risk free swap curve is used for discounting swaps. This is because the 3-month zero-coupon risk free swap curve is the most liquid zero-coupon risk free swap curve in the South African market. The rates used to derive the 3-month zero-coupon risk free swap curve consist of credit risk inherent in a loan for a duration of three months.

There has been a shift in global markets to using an Overnight Indexed Swap (OIS) zero-coupon curve instead of the interbank deposit rate swap curve, for discounting swaps, this is due to the additional credit risk that these interbank deposit rate curves contain and because these curves are not completely risk free (Gregory, 2012:284). The rates used to derive the OIS zero-coupon curve consists only of credit risk on a loan that has an overnight duration, which means the credit risk is negligible when using this curve (Koers, 2011:13).

The usage of the OIS zero-coupon curve for discounting swaps has lead to new market practices in the valuation of these derivatives. The single-curve approach to swap valuation, where one curve is used to calculate the forward and discount rates, has been changed by the financial crisis (Koers, 2011:1). The relatively new multi-curve approach to swap valuation, where one curve is used to calculate the forward rates and another (OIS zero-coupon curve) is used to determine the discount rates, has become the norm in most of the developed countries (Gregory, 2012:284). A swap can be valued using these different approaches. This is of significance as it illustrates how important the choice of valuation approach is and because swap values can differ for each approach.

In global markets, the OIS zero-coupon rate is a proxy for the risk free rate in discounting swaps. The OIS zero-coupon rate for a particular maturity is obtained by reading it off from the term structure of the OIS zero-coupon curve. In the UK, the OIS zero-coupon rate is used as a proxy for the risk free rate rather than the X-month zero-coupon risk free swap rate. For example, if a swap has payments every three months, then the 3-month zero-coupon risk free swap curve would be used for calculation of forward rates and the OIS zero-coupon curve would be used for discounting. This is referred to as the multi-curve approach to swap valuation.

In South Africa, entities use the 3-month zero-coupon risk free swap curve as a proxy for the risk free curve to swaps because an overnight indexed swap market does not exist. Thus, for a swap with payments every three months the 3-month zero-coupon risk free swap curve is used for both forwarding and discounting which is referred to as the single curve approach to swap valuation. Furthermore, there are a number of different ways in which swap curves can be constructed. This leads to complications because different entities will use differently constructed zero coupon risk free swap curves to determine the risk free valuation of swaps.

1.1 PROBLEM STATEMENT

Fair value measurement of swaps in a risk neutral world is complicated because of the difference in proxies for the zero-coupon risk free discounting curve. The movement in developed markets from the single-curve to the multi-curve swap valuation approach has had a significant impact on swap valuation.

According to Gregory (2012:286), the proxy for the risk free discount curve has changed to the OIS zero-coupon curve in developed markets, however the South African market still uses the 3-month zero-coupon risk free swap curve along with the single-curve swap valuation approach. Constructing the 3-month zero-coupon risk free swap curve in different ways would result in different values for the same swap.

1.2 RESEARCH QUESTION

This research assignment attempts to address the following question:

What is the impact on the value of a swap in the risk neutral world when differently constructed curves are used to value the same swap, along with the different swap valuation approaches?

1.3 RESEARCH OBJECTIVES

It is the aim of this research assignment to develop an understanding of how to construct certain zero-coupon risk free swap curves in global markets. However, the research will primarily attempt to determine how the fair valuations of swaps in the risk neutral world are influenced when different risk free curves are used for discounting, along with the different swap valuation approaches.

In the South African market, different methodologies for the construction of the 3-month zero-coupon risk free swap curve will be presented and explained. This will be done along with an explanation of the impact that these different methodologies have on the fair valuation of swaps in the risk neutral world.

In the UK market, the fair value of swaps in the risk neutral world will be determined using both the single-curve and multi-curve swap valuation approaches. These valuations will be compared along with the spreads of the different curves that are used to value swaps. This is implemented to illustrate how the value of a swap will differ, in the risk neutral world, when these different approaches for swap valuation are used.

1.4 IMPORTANCE / BENEFITS OF THE STUDY

This research assignment will outline how fair value measurements of swaps differ in a risk neutral world when using different curves as a proxy for the risk free rate. This will be of importance to entities with limited resources and accounting capabilities that hold positions in these instruments, because these instruments are fair valued using these curves.

In the South African market, the 3-month zero-coupon risk free swap curve is used as a proxy for the risk free rate when discounting swaps. Thus, different ways of constructing the 3-month zero-coupon risk free swap curve and the impact this has on the fair valuation of these swaps will be important.

In the UK market, an understanding of the difference in fair value of swaps, in the risk neutral world, when using both the single-curve and multi-curve approaches to swap valuation is of significance. These differences could highlight how markets, that use the single-curve approach, could possibly value swaps incorrectly.

1.5 RESEARCH DESIGN AND METHODOLOGY

The research assignment will be divided into two parts. The first part of the research assignment is implemented in the South African market. In this part, an explanation with the algorithms of different interpolation methodologies for the construction of the 3-month zero-coupon risk free swap curve will be presented, along with how the fair valuations of swaps differ in the risk neutral world when using these different methodologies for curve construction. The different interpolation methodologies that will be considered include: linear interpolation, log linear interpolation and monotone preserving $r(\tau)\tau$ interpolation. South African market data available on 30 June 2014 for zero-coupon swap rates will be considered.

The second part is based on the UK market. This part of the research assignment will analyse the spreads between the different curves used as a proxy for the risk free curve and will determine the sensitivity in the risk neutral valuation of swaps by using these different discount curves in the single-curve approach followed by the multi-curve approach. The curves that will be used in the analysis include: OIS zero-coupon curve in the UK, GBP 3-month zero-coupon swap curve and the GBP 6-month zero-coupon swap curve. The data that will be considered for this part is obtained on 11 September 2014.

1.6 CHAPTER OUTLINE

This section provides the reader with an overview of what to expect. The following chapter provides a literature review where all previous research used in this assignment is discussed and explained. The literature includes the methodology behind zero-coupon swap curve construction, interpolation methods and the bootstrap algorithm.

Chapter 3, outlines how the 3-month zero-coupon swap curve for the South African market can be constructed using different methods of interpolation. The interpolation methods include linear, log-linear and monotone preserving $r(\tau)\tau$ interpolation. This chapter also presents the implementation of swap valuation using the single-curve approach in the South-African market along with results of how the value of a swap differs when using the differently constructed curves to value the same swap. A summary of the findings will be presented at the end of this chapter.

The next chapter applies both the single-curve and multi-curve approaches to swap valuation in the UK market. The differences in swap values obtained are presented, explained and analysed. A summary of the findings is presented at the end of this chapter. The last chapter gives a conclusion, as well as the limitations of this study.

CHAPTER 2
LITERATURE REVIEW

In this chapter, literature in the field of yield curve construction and swap valuation is revisited. Firstly, zero-coupon swap curve construction and the details thereof will be discussed. Interpolation methods along with their features will form the topic of the second section. The third section introduces the bootstrap algorithm used for swap curve construction. In the last section, literature on the valuation of interest rate swaps is discussed. The literature review provides the theory needed for constructing the zero-coupon risk free swap curve and using the single- and multi-curve approaches for swap valuation.

2.1 ZERO-COUPON SWAP CURVE CONSTRUCTION

A zero-coupon swap curve depicts the relationship between the zero-coupon swap rates and the term structure. A zero-coupon swap curve is used to determine the present value of a swap in the risk neutral world. A zero-coupon swap curve consists of observed market interest rates derived from market instruments that are the most liquid and dominant for their time horizons. The observed market rates are bootstrapped and combined with an interpolation algorithm to form the zero-coupon swap curve. This section describes the methodology for constructing a zero-coupon swap curve.

2.1.1 Zero-coupon swap curve inputs

The inputs for the zero-coupon swap curve should cover the complete term structure of the curve (i.e., short-, middle-, and long-term parts). The inputs consist of interbank deposit rates, forward rate agreements (FRAs) and swap rates all quoted in the market. A zero-coupon swap curve should be constructed using inputs that consist of similar credit properties, a high degree of liquidity and keeping in mind that these inputs are currency-dependant (see Appendix 1 for a description of the different currency related inputs).

The technique that is used for constructing the zero-coupon swap curve is to divide the term structure into three term brackets (Ron, 2000:11). The short end of the zero-coupon swap curve, out to three months is derived using interbank deposit rates. The middle area of the zero-coupon swap curve, up to two years is derived from FRAs. The long end of the zero-coupon swap curve, out to thirty years is derived directly from swap rates (observable coupon swap rates).

2.1.2 Deriving a zero-coupon swap curve

The observed market interest rates (inputs) are combined with different interpolation techniques to derive a zero-coupon swap curve for a relevant tenor. The zero-coupon swap curve at different dates is constructed using the relevant settlement day convention (business-day or modified following business-day). It is important to note that the yield day count convention and the payment frequency for the zero-coupon swap curve differs by currency.

2.1.3 The short end of the zero-coupon swap curve

As proposed by Ron (2000), the short end of the zero-coupon swap curve out to three months is constructed using the overnight, one-month and three-month deposit rates. These short end deposit rates are inherently zero-coupon rates, which need to be converted to the base currency swap rate compounding frequency and relevant day count convention. The following equation is solved to compute the continuously compounded zero-coupon swap rate (r_c):

$$\mathrm{r}_c = \frac{t_y}{t_m} \times ln\left(1 + \frac{r_d}{\frac{t_y}{t_m}}\right) \tag{2.1}$$

Where,

$n =$ Number of years;

$r_d =$ Observed market deposit rate;

$\mathrm{t}_y =$ Number of days in a year specified according to the day count convention used;

$t_m =$ Number of days to maturity.

2.1.4 The middle area of the zero-coupon swap curve

The middle area of the zero-coupon swap curve out to two years is derived from either FRA rates or interest rate futures observed in the market. This paper will only consider FRAs. The FRAs in combination with the calculated short end of the zero-coupon swap curve along with the same interpolation algorithm forms the middle area of the curve.

Forward interest rates are the rates of interest implied by current zero-coupon swap rates for periods of time in the future. Typical examples of observed FRA rates used in the construction of the zero-coupon swap curve are the following: 1x4, 2x5, 3x6, 4x7, 5x8, 6x9, 7x10, 8x11, 9x12, 12x15, 15x18, 18x21 and 21x24. Here are two examples of the interpretation of such forward rates:

- A 1x4 FRA is the three-month forward rate in one month as seen from today.
- A 5x8 FRA is the three-month forward rate in five months time as seen from today.

To calculate zero-coupon swap rates from forward rates the following method is used (Hull, 2012:85). Assume that the rates are continuously compounded and that time periods are i.t.o. day count conventions.

$$R_F = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$

$$\rightarrow \; R_2 = \frac{R_F(T_2 - T_1) + R_1T_1}{T_2} \qquad (2.2)$$

Where,

$R_F =$Known forward interest rate for the period of time between time T_1 and T_2;

$R_1 =$ Known zero-coupon swap rate for maturity T_1;

$R_2 =$ Unknown zero-coupon swap rate for maturity T_2.

2.1.5 The long end of the zero-coupon swap curve

The long end of the swap curve out to thirty years is derived directly from observed swap par rates. These are “plain vanilla” interest rate swaps with fixed rates exchanged for floating interest rates. The swap par rates are compounded semi-annually in most markets (Ron 2000:6). The bootstrap method is used along with an interpolation algorithm to derive the swap rates from the swap par rates and will be explained in the following sections.

2.2 INTERPOLATION METHODS

Interpolation is a method of constructing new data points within the range of a discrete set of known data points (called knot points) (Du Preez, 2011:24).

The inputs for an interpolation method, used to interpolate yield curve data, are a set of zero-coupon spot rates $r_1 < r_2 \ldots < r_n$ with corresponding times $\tau_1 < \tau_2 \ldots < \tau_n$ and returns a piecewise continuous function. The interpolation function $r(\tau)$ allows us to find the zero-coupon swap rate corresponding to any point in time τ, where $\tau_1 \leq \tau \leq \tau_n$. An interpolation function must satisfy the following conditions:

- It must reproduce the input set of zero-coupon swap rates, this implies that $r(\tau_i) = r_i$ for $i = 1,2, \ldots, n$.
- It must produce a piecewise continuous spot rate function, thus $\lim_{\delta \to \infty} r(\tau_i + \delta) = \lim_{\delta \to \infty} r(\tau_i - \delta)$, for $i = 1,2, \ldots, n$.

The ideal interpolation function would be one that satisfies the above constraints and produces a forward rate curve that is both continuous and positive. From an economic point of view discontinuous forward rates do not make sense and negative forward rates imply arbitrage opportunities (Du Preez, 2011:24).

Interpolation methods are needed for deriving the full term structure of the swap curve. It is important that the interpolation algorithm and the bootstrap methodology are intimately connected (Hagan, West, 2006:4). In this section various interpolation methods will be described.

2.2.1 Linear interpolation of zero-coupon swap rates

Suppose $\tau \in (\tau_1, \tau_n)$ is given and that τ is not equal to any of the τ_i. Determine i such that $\tau_i < \tau < \tau_{i+1}$. This method only uses the zero-coupon swap rates r_i and r_{i+1} in order to estimate $r(\tau)$.

$$r(\tau) = \frac{\tau-\tau_i}{\tau_{i+1}-\tau_i} r_{i+1} + \frac{\tau_{i+1}-\tau}{\tau_{i+1}-\tau_i} r_i \tag{2.3}$$

Hagan and West (2006:95) show that this interpolation method does not produce continuous forward rates, which means that this method is not optimal for swap curve construction.

2.2.2 Linear on the logarithm of zero-coupon swap rates

This method is called log-linear or sometimes even exponential interpolation. Suppose $\tau \in (\tau_1, \tau_n)$ is given and that τ is not equal to any of the τ_i. Determine i such that $\tau_i < \tau < \tau_{i+1}$. This method only uses the zero-coupon swap rates r_i and r_{i+1} in order to estimate $r(\tau)$.

If,

$$\ln r(\tau) = \frac{\tau-\tau_i}{\tau_{i+1}-\tau_i} \ln r_{i+1} + \frac{\tau_{i+1}-\tau}{\tau_{i+1}-\tau_i} \ln r_i$$

Then it implies that,

$$r(\tau) = r_{i+1}^{\frac{\tau-\tau_i}{\tau_{i+1}-\tau_i}} \times r_i^{\frac{\tau_{i+1}-\tau}{\tau_{i+1}-\tau_i}} \tag{2.4}$$

Hagan and West (2006:96), shows that this method does not guarantee positive forward rates. The two interpolation methods above are the most popular methods but have continuity difficulties associated with them. Thus, they should not be used for anything but naive interpolation of yield curves as advised by the above-mentioned authors.

2.2.3 Monotone preserving $r(\tau)\tau$ interpolation

In this method, a shape preserving cubic Hermite method of interpolation is applied to the log capitalisation function, which is called the monotone preserving $r(\tau)\tau$ method. Consider the set of rates (knot points) $r_1, r_2, \dots, r_n$ for maturities $\tau_1, \tau_2, \dots, \tau_n$. A yield curve function $r(\tau)$ is established for $\tau \in [\tau_1, \tau_n]$.

Consider the interpolant:

$$r(\tau)\tau = a_i + b_i(\tau - \tau_i) + c_i(\tau - \tau_i)^2 + d_i(\tau - \tau_i)^3 \qquad for\ \ \tau_i \le \tau \le \tau_{i+1}$$

and define

$$h_i = \tau_{i+1} - \tau_i$$

$$m_i = \frac{r_{i+1} \cdot \tau_{i+1} - r_i \cdot \tau_i}{h_i} \qquad for\ \ i = 0,1,2,\dots,n-1$$

Assume the instantaneous set of forward rates $f_1, f_2, \dots, f_n$ for the maturities $\tau_1, \tau_2, \dots, \tau_n$ are known a priori. Then Hagan and West (2006) show that:

$$a_i = r_i\tau_i$$

$$b_i = f_i$$

$$c_i = \frac{3m_i - f_{i+1} - 2f_i}{h_i}$$

$$d_i = \frac{f_{i+1} + f_i - 2m_i}{h_i^2} \qquad for\ \ i = 1,2,\dots,n-1$$

In practice, instantaneous forward rates are not necessarily observable and need to be estimated. Hagan and West (2006) propose estimating $f_i,\ for\, i = 2,3,\dots,n-1$ as the slope at τ_i, of the quadratic that passes through the point $\left(\tau_{i-j}, m_{i-j-1}\right),\ for\, j = 1,0,-1$. The instantaneous forward rates at the end points, i.e. f_1 and f_n are chosen to ensure that $f_1' = f_2' = 0$. The instantaneous forward rates are, thus, estimated as:

$$f_i = \frac{(\tau_{i-1} - \tau_i)m_i}{\tau_{i+1} - \tau_{i-1}} + \frac{(\tau_{i+1} - \tau_i)m_{i-1}}{\tau_{i+1} - \tau_{i-1}} \qquad for\ \ i = 2,3,\dots,n-1$$

and

$$f_1 = r_1$$

$$f_n = m_{n-1}$$

The log capitalisation function $(r(\tau)\tau)$ need be monotone increasing. Thus, impose the following condition proposed by de Boor & Swartz (1977) to ensure the function implies a positive forward curve:

If $r(\tau)\tau$ *is locally increasing at* τ_i *and if* $f_i \le \min(f_i, 3\,min(m_i, m_{i-1}))$

then $r(\tau)\tau$ *will be monotone increasing on the interval* $[\tau_2, \tau_{n-1}],\ for\ \ i = 1,2,\dots,n-1.$

The monotone preserving $r(\tau)\tau$ interpolation method is continuous, monotone increasing and ensures positive forward rates. This means that this interpolation method can be used for swap curve construction. In the following section the bootstrap algorithm for zero-coupon swap curve construction is discussed.

2.3 BOOTSTRAP ALGORITHM

The bootstrap algorithm is used to determine the full term structure of the zero-coupon swap curve. In Figure 1, a formula with an illustration of the bootstrap procedure is presented. The rates $Z1$, $Z2$ and $Z3$ are the known zero-coupon risk free swap rates for known times $T1$, $T2$ and $T3$. The swap par rate for a swap that has N payments per year at time $T4$ is SPR_{T4}. This swap par rate is observed in the market. The unknown zero-coupon risk free swap rate $Z4$ for the known time $T4$ is determined by substituting the known values into the equation and solving for $Z4$.

$$100 = SPR_{T4} \cdot \frac{1}{N} \cdot 100\left(e^{-Z1\times T1} + e^{-Z2\times T2} + e^{-Z3\times T3} + e^{-Z4\times T4}\right) + 100e^{-Z4\times T4}$$

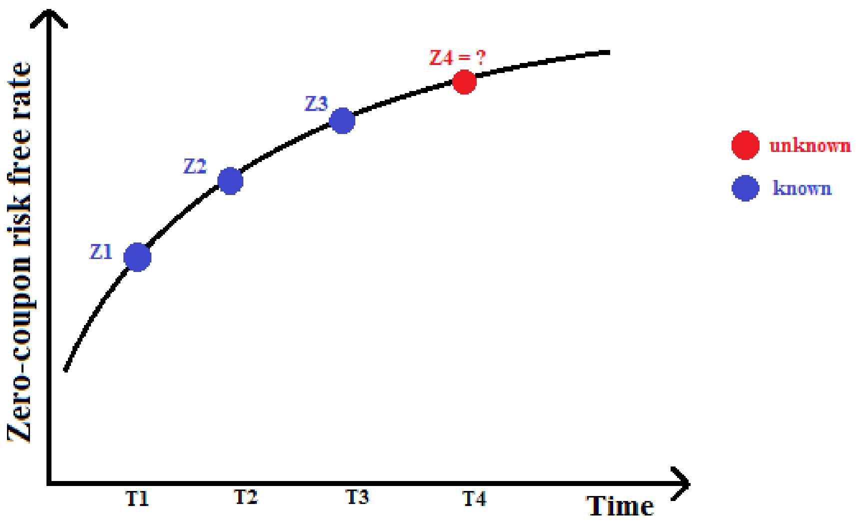

Figure 1: An illustration of the bootstrap procedure

A clarified in depth version of the bootstrap algorithm, given in Hagan and West (2006), is discussed in this section. The observed instruments available for the bootstrap algorithm are LIBOR-type instruments, FRAs, and swaps. Assume that the LIBOR instruments along with the FRAs expire before the swaps.

The algorithm to bootstrap a zero-coupon swap curve follows as:

i) Arrange the instruments in order of expiry term in years. Let these expiry terms be $\tau_1, \tau_2, \dots, \tau_N$.

ii) Find the continuously compounded rates corresponding to the LIBOR instruments using equation (2.1).

iii) Create a first estimate curve. For example, for each of the FRA and swap instruments let r_i be the market rate of the first instrument.

iv) (*) A first estimate curve is available with terms $\tau_1, \tau_2, \dots, \tau_N$ and rates $r_1, r_2, \dots, r_N$. These values are passed to the interpolator algorithm.

v) Update the FRA estimates for it to have zero value when the FRA is dealt:

$$C(t, t_1)(1 + \alpha f) = C(t, t_2) \qquad (2.5.1)$$

Here, t_1 is the settlement date of the FRA, t_2 is the expiry date (thus, it is a $t_1 \times t_2$ FRA), f is the FRA rate, and $\alpha = t_2 - t_1$ the period of the FRA, taking into account the relevant day count convention. Note that the FRA rate is quoted as a simple rate and $C(t, t_n) = e^{r(t_n) \times \tau_n}$, where $\tau_n = t_n - t$. Equation (2.5.1) can be written as:

$$r(t_2) = \frac{1}{\tau_2}[C(t, t_1) + \ln(1 + \alpha f)] \qquad (2.5.2)$$

This gives the required iterative formula for the bootstrap: $C(t, t_1)$ is found by reading off (interpolating) the estimated curve. The term that emerges on the left is noted for the next iteration.

vi) For a swap to have zero value when it is dealt:

$$R_n \sum_{i=1}^{n} \alpha_i Z(t, t_i) = 1 - Z(t, t_n)$$

Where, R_n is the swap par rate, α_i is the time in years from t_{i-1} to t_i calculated with the relevant day count convention and $Z(t, t_i)$ is the discount factor from time t_i back to time t. The equation can be rearranged such that,

$$Z(t, t_n) = \frac{1 - R_n \sum_{i=1}^{n-1} \alpha_i Z(t, t_i)}{1 + R_n \alpha_n} \qquad (2.5.3)$$

where,

$$Z(t, t_n) = e^{-r(t_n) \times t_n}.$$

Here t_n is the time in years from time t to time $\mathrm{t_n}$ calculated with the relevant day count convention and $r(t_n)$ is the continuously compounded zero-coupon rate for time t_n.

Thus, equation (2.5.3) can be rewritten as

$$r(t_n) = \frac{-1}{t_n} \times ln\left[\frac{1 - R_n \sum_{j=1}^{n-1} \alpha_j Z(t, t_j)}{1 + R_n \alpha_n}\right]. \quad (2.5.4)$$

Which gives the required iterative formula for the bootstrap: again all terms on the right are found by interpolating the estimated curve, the term on the left is noted for the next iteration.

vii) This is done for all instruments. Now return to (*) and iterate until convergence. Convergence to double precision is recommended; this will occur in approximately 10-20 iterations for favoured methods of interpolation.

2.4 SWAP VALUATION

Looking back, the curves constructed by applying the previously described interpolation and bootstrap algorithm will be used in the swap valuation process. A swap is an agreement between two entities to exchange cash flows in the future. A swap is an over-the-counter derivative, which defines the cash flows to be paid and the way in which they are to be calculated. The focus will be on “plain vanilla” interest rate swaps.

2.4.1 Plain vanilla interest rate swaps

In a “plain vanilla” interest rate swap, a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principle for a predetermined number of years. In return, it receives interest at a floating rate, usually LIBOR, on the same notional principle for the same period of time.

The value of a swap at initiation is close to zero. The value of the swap changes through time and can become positive or negative. Note that the first floating rate of such a swap is known at the initiation of the swap, because the prevailing floating rate is used to determine the floating payment at each payment date. The notional principle of a swap is not exchanged at the end of the swap. The swap par rate is the fixed rate of the swap.

From here on forward referring to a swap, implies the reference to a “plain vanilla” interest rate swap. The valuation of swaps will be discussed in the following sections.

2.4.2 The principles underlying swap valuation

The fixed rate, floating rate, payment frequency, business day convention, nominal principle and length of a swap is specified at the initiation of a swap. A swap payment can only occur on a business day according to the business day convention.

The value of a swap is determined as $\sum_{t=1}^{n} PV(NCF_t)$, which is the sum of the present values of each future net cash flow at a payment date over the remaining life of a swap, where NCF_t is the net cash flow of the swap at a time in the future t. Each future net cash flow consists of a floating rate cash flow and a fixed rate cash flow. If the swap is valued for the floating rate payer, each net cash flow at a future payment date t is determined as $NCF_t = FixedCF_t - FloatCF_t$. Whereas each net cash flow for the fixed rate payer at a future payment date t is determined as $NCF_t = FloatCF_t - FixedCF_t$.

The fixed rate cash flow of a swap at a future date t is determined as $FixedCF_t = NP \times FixR \times \tau_t$ where NP is the nominal principle of a swap, $FixR$ is the fixed rate of the swap quoted as a par rate and τ_t is the time until the payment t is due, measured as a fraction of a year using the relevant day count convention. The floating rate cash flow at a future date t of a swap is determined as $FloatCF_t = NP \times FloatR_t \times \tau_t$ where $FloatR_t$ is the relevant floating rate for time t quoted as a par rate.

2.4.3 The single-curve approach to swap valuation

A swap can be characterised as a portfolio of forward rate agreements (FRAs) as proposed by Hull (2012:162). A FRA can be valued assuming that the forward rates are realised. The procedure for the valuation of swaps in terms of FRAs follows broadly as:

i) Use the zero-coupon swap curve to calculate forward rates for each of the LIBOR rates that will determine swap cash flows.
ii) Calculate the swap cash flows on the assumption that the LIBOR rates will equal the forward rates.
iii) Discount these swap cash flows using the zero-coupon swap curve to obtain the swap value.

As mentioned, a swap is worth close to zero initially. This does not mean that each FRA will be worth close to zero when the swap is initiated, but rather that the sum of the FRAs will be close to zero. In general, some FRAs will have negative values and some will have positive values. This is also known as the single-curve approach to swap valuation mainly because the forward and discount rates are determined by a single curve (zero-coupon swap curve).

2.4.4 The multi-curve approach to swap valuation

The following has been adapted from Hull (2012:162). As with the single-curve approach to swap valuation, a FRA can be valued assuming that the forward rates are realised. The procedure for the multi-curve valuation of swaps in terms of FRAs follows broadly as:

i) Use the relevant tenor zero-coupon swap curve to calculate forward rates for each of the LIBOR rates that will determine swap cash flows.
ii) Calculate the swap cash flows on the assumption that the LIBOR rates will equal the forward rates.
iii) Discount these swap cash flows using the OIS zero-coupon curve to obtain the swap value.

This is also known as the multi-curve approach to swap valuation mainly because the forward rates are determined by one curve (zero-coupon swap curve) and discount rates are determined by another (OIS zero-coupon swap curve).

2.5 OBJECTIVES

In the next chapter this literature review concerning swap curve construction along with the single-curve approach to swap valuation will be applied to the South African market. In Chapter 4, the literature on the valuation of swaps along with the single-curve and multi-curve approaches to swap valuation will be applied to the UK market.

2.6 SUMMARY

The literature review provided the needed theory on swap curve construction and swap valuation such that it could be implemented in various markets. In this chapter, swap curve construction along with different interpolation methods were discussed. A single bootstrap algorithm for a swap curve and the valuation of swaps had also been discussed in this chapter. This literature review was concluded with a discussion of the objectives on how the literature review will be applied to both the South African and UK markets. The next two chapters give both the results obtained from applying the literature to these markets and methods of applying this literature in these markets.

CHAPTER 3

ZERO-COUPON SWAP CURVE CONSTRUCTION AND SINGLE-CURVE SWAP VALUATION IN THE SOUTH AFRICAN MARKET

The literature discussed in the previous chapter will be applied throughout this chapter. This chapter will present the construction of the 3-month zero-coupon risk free swap curve in the South African market using various interpolation methods. The valuation of swaps using these differently constructed zero-coupon risk free swap curves along with the single-curve approach to swap valuation will also be presented.

Firstly, the procedure for constructing the 3-month zero-coupon risk free swap curve in the South African market using monotone preserving $r(\tau)\tau$ interpolation is discussed. In section two, the procedure for constructing the zero-coupon risk free swap curve using linear interpolation is discussed. The third section will present the procedure for the construction of the zero-coupon risk free swap curve using log linear interpolation. In the fourth section, a single swap with varying time lengths and swap par rates will be valued, using these differently constructed zero-coupon risk free swap curves, and the possible errors in swap value will be presented. A conclusion to the chapter is presented in the last section.

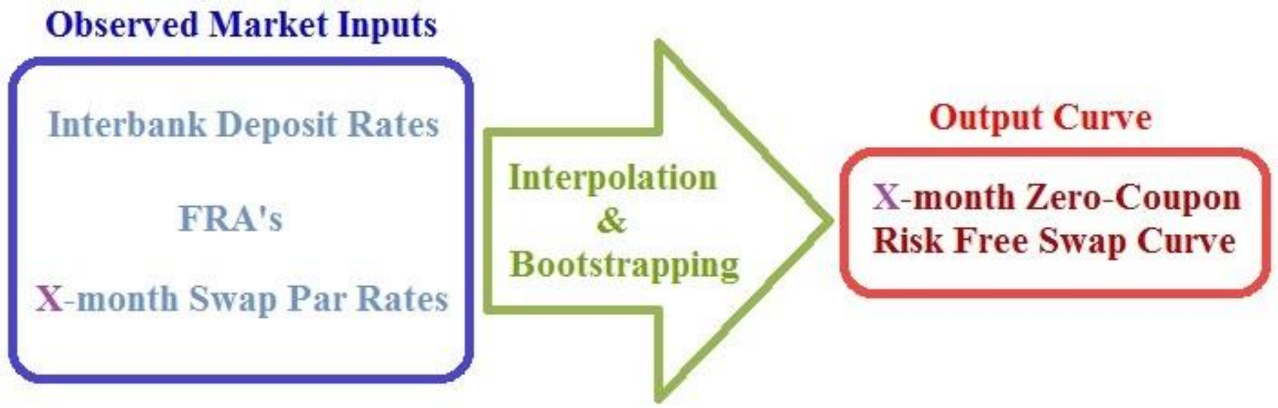

Figure 2: Process for constructing the X-month zero-coupon risk free swap curve

The process for constructing an X-month zero-coupon risk free swap curve is presented in Figure 2. This process should be kept in mind when going through this chapter. The observed market inputs are interpolated and bootstrapped to obtain the 3-month zero-coupon risk free swap curve. Note that if 3-month swap par rates are passed through the interpolation and bootstrapping as inputs then the 3-month zero coupon risk free curve will be determined as the output curve. In the next section the construction of the 3-month zero-coupon risk free swap curve using monotone preserving $r(\tau)\tau$ interpolation is presented.

3.1 3-MONTH ZERO-COUPON SWAP CURVE CONSTRUCTION USING MONOTONE PRESERVING $r(\tau)\tau$ INTERPOLATION

The 3-month zero-coupon risk free swap curve consists of observed market interest rates derived from market instruments that are the most liquid and dominant for their time horizons. The observed market rates are bootstrapped and combined with an interpolation algorithm to form the 3-month zero-coupon risk free swap curve. In this section the step wise procedure for constructing the 3-month zero-coupon risk free swap curve using monotone preserving $r(\tau)\tau$ interpolation is discussed.

3.1.1 Step 1: Determine the inputs

In this step the inputs for the zero-coupon risk free swap curve is determined. The inputs for the zero-coupon risk free swap curve should cover the complete term structure of the curve including the short-, middle-, and long-term parts. Suppose a 3-month zero-coupon risk free swap curve with a term structure of thirty years is to be constructed. An example of the inputs used in the construction of the zero-coupon risk free swap curve is presented in Table 1. The inputs consist of interbank deposit rates, forward rate agreements (FRAs) and swap rates all quoted in the market.

The term structure of the zero-coupon risk free swap curve is divided into three term brackets. The short end of the curve, up to three months is derived using interbank deposit rates. The middle area of the curve, up to twenty one month's is derived from FRAs if the inputs are as in Table 1. The long end of the zero-coupon risk free swap curve, out to thirty years is derived directly from swap par rates which are observable in the market.

Table 1: Example of inputs for the construction of the zero-coupon risk free swap curve

Swap Curve	
Code	MTM
SAFEX Overnight	5.290
1m JIBAR	5.725
3m JIBAR	5.825

Interbank deposit rates

Swap Curve	
Code	MTM
1x4	6.060
2x5	6.107
3x6	6.270
4x7	6.323
5x8	6.463
6x9	6.517
7x10	6.660
8x11	6.707
9x12	6.847
12x15	7.013
15x18	7.220
18x21	7.367

FRAs

Swap Curve	
Code	MTM
2y	6.801
3y	7.110
4y	7.348
5y	7.540
6y	7.711
7y	7.858
8y	7.994
9y	8.113
10y	8.202
12y	8.367
15y	8.533
20y	8.603
25y	8.560
30y	8.493

Swap par rates

Source: Johannesburg Stock Exchange on 30/06/2014

3.1.2 Step 2: Determine the yield calculation conventions

In this step, it is necessary to determine the day count conventions of the South African currency. The day count convention for the ZAR is Actual/365 and settlement dates are chosen using the modified following business-day convention (SARB, 2013:10).

The modified following business-day convention is frequently used when constructing swaps, where the cash flow occurs on the next business day unless that day falls in a different month. In this case, the cash flow occurs on the immediately preceding business day to keep payments in the same month (ISDA 1999).

3.1.3 Step 3: The short end knot points of the curve

The short end knot points of the 3-month zero-coupon risk free swap curve out to three months of the thirty year term structure is determined using the SAFEX Overnight, 1m JIBAR and 3m JIBAR deposit rates. These short end deposit rates are inherently zero-coupon risk free rates, which are quoted as simple rates that need be converted to continuously compounded rates using the relevant day count convention.

The following equation is solved to compute the continuously compounded zero-coupon risk free swap rate (δ_n):

$$\left(1 + i_n \times \frac{n}{d}\right) = e^{\delta_n \times \frac{n}{d}}$$

$$\rightarrow \delta_n = \frac{d}{n}\left(1 + i_n \times \frac{n}{d}\right) \tag{3.1}$$

Where

$n =$ number of days until deposit rate is due,

$i_n =$ the observed market deposit rate due in n days,

$\delta_n =$ the continuously compounded zero-coupon risk free swap rate due in n days,

$d = 365$, which is the number of days in a year according to the South-African day count convention.

Using the above formula, the 1-day, 1-month and 3-month zero-coupon risk free swap rates are determined and they are called the knot points for the short end of the curve. Note that the next step is not interpolating, this is because the monotone preserving $r(\tau)\tau$ interpolation method is a cubic function which requires one knot point before and two knot points after the zero-coupon risk free rate that is interpolated. Thus, interpolating will only give the correct zero-coupon risk free swap rates for the days between the 1-day and 1-month knot points, which is not the full short end of the curve.

3.1.4 Step 4: Determine the middle area knot points from the short end knot points

In this step, a few but not all middle area knot points are determined from the short end knot points, which were determined from the previous step, along with the observed FRA rates. The observed FRA rates used in this step are 1x4, 3x6, 4x7, 6x9, 7x10, 9x12, 12x15, 15x18 and 18x21. Note that the 2x5, 5x8 and 8x11 FRA rates cannot be used because the 2-month zero-coupon risk free swap rate has not been determined yet.

The FRAs are quoted as simple rates and need to be converted to continuously compounded interest rates by using equation (3.1). The zero-coupon risk free swap rates are calculated from the continuously compounded FRA rates along with the calculated short end knot points of the zero-coupon risk free swap curve by using the following formula:

$$e^{\tau_n \times \delta_n} \times e^{(\tau_p - \tau_n) \times f_{n \times p}} = e^{\tau_p \times \delta_p}$$

$$\rightarrow \frac{1}{\tau_p} \ln\left(e^{\tau_n \times \delta_n} \times e^{(\tau_p - \tau_n) \times f_{n \times p}}\right) = \delta_p$$

$$\rightarrow \frac{1}{\tau_p}\left(\{\tau_n \times \delta_n\} + \left\{(\tau_p - \tau_n) \times f_{n \times p}\right\}\right) = \delta_p \tag{3.2}$$

Where

$\delta_i =$ the zero-coupon swap rate for time i,

$\tau_i =$ the time period for when the rate δ_i is due measured in years,

$f_{i \times j} =$ the continuously compounded forward rate for time τ_i that lasts up to time τ_j.

This formula is equivalent to equation (2.2) except that the FRA rate is a continuously compounded rate in this case. The zero-coupon risk free swap rates obtained from the FRAs are the 4-, 6-, 7-, 9-, 10-, 12-, 15-, 18- and 21-month zero-coupon risk free swap rates, and are also called knot points. Note that the 5-, 8- and 11-month zero-coupon risk free swap rates need to be determined, but this can only be done if the 2-month zero-coupon risk free swap rate is known.

3.1.5 Step 5: Interpolating the short end of the curve

The full term structure up to three months of the zero-coupon risk free swap curve can be determined by interpolating. The knot points used as inputs for interpolation are the 1-day, 1-month, 3-month and 4-month zero-coupon risk free swap rates that were determined in the previous steps. The monotone preserving $r(\tau)\tau$ interpolation method is implemented in the following procedural way:

i) For each knot point $i = 1,2,\ldots,n$, define τ_i as the time to maturity of the i^{th} zero-coupon risk free swap rate and r_i as the zero-coupon risk free swap rate relating to τ_i. Set $\tau_0 = 0$.
ii) For $i = 0,1,2,\ldots,n-1$ set up:

$$h_i = \tau_{i+1} - \tau_i$$

and,

$$m_i = \frac{r_{i+1} \cdot \tau_{i+1} - r_i \cdot \tau_i}{h_i}$$

Set $m_n = m_{n-1}$.

iii) For $i = 0,1,2, \dots, n$ set up:

$$a_i = r_i \tau_i$$

iv) For $i = 2,3, \dots, n-1$ set up:

$$b_i = min\left\{\frac{(\tau_i - \tau_{i-1})m_i + (\tau_{i+1} - \tau_i)m_{i-1}}{\tau_{i+1} - \tau_{i-1}}, 3\,min(m_{i-1}, m_i)\right\}$$

Set $b_1 = b_0 = m_0$ along with $b_n = m_n$.

v) For $i = 1,2, \dots, n-1$ set up:

$$c_i = \frac{3m_i - b_{i+1} - 2b_i}{h_i}$$

Set $c_0 = c_n = 0$.

vi) For $i = 1,2, \dots, n$ set up:

$$d_i = \frac{b_{i+1} + b_i - 2m_i}{h_i^2}$$

Set $d_0 = 0$.

vii) Finally, for the zero-coupon risk free swap rate due at time t, find where it fits relative to the maturities of the knot points, so that $\tau_i \leq t \leq \tau_{i+1}$. To find the zero-coupon risk free swap rate due at time t, use the formula:

$$r(t) = \frac{a_i + b_i(\tau - \tau_i) + c_i(\tau - \tau_i)^2 + d_i(\tau - \tau_i)^3}{t}$$

Notice that no $f_i's$ are declared here and for simplicity $b_i's$ were declared and used. Using the above procedure the zero-coupon risk free swap rate for each day between the 1-day, 1-month and 3-month knot points can be determined. This concludes the construction of the short end of the zero-coupon risk free swap curve out to three months of the thirty year term structure. Note that the 2-month zero-coupon risk free swap rate has been determined in this process.

3.1.6 Step 6: Determine the remaining middle area knot points

In this step, the rest of the middle area knot points are determined from the 2-month zero-coupon risk free swap rate along with the rest of the observed FRAs. The observed FRA rates used in this step are 2x5, 5x8 and 8x11. Use equation (3.1) to convert the observed FRA rates then use equation (3.2) to determine the 5-, 8- and 11-month zero-coupon risk free swap rates. Now the knot points have been determined for the 1-day, 1-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 11-, 12-, 15-, 18- and 21-month maturities.

3.1.7 Step 7: Interpolate the middle area of the zero-coupon risk free swap curve

This step uses all the knot points determined up to now along with the monotone preserving r(τ)τ interpolation method to determine the middle area of the zero-coupon risk free swap curve up to 18-months. Note that the interpolation method needs the 2-year zero-coupon risk free swap rate to determine the zero-coupon risk free swap curve up to 21-months.

Use the exact same interpolation procedure as in Step 5 except now the knot points used are the 1-day, 1-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 11-, 12-, 15-, 18- and 21-month zero-coupon risk free swap rates. The zero-coupon risk free swap curve is now determined for each day up to 18-months of the thirty year term structure.

3.1.8 Step 8: Bootstrapping the long end of the zero-coupon risk free swap curve

In this step the knot points for the long end of the zero-coupon risk free swap curve is determined by bootstrapping. Figure 1, in Section 2.3 should be kept in mind throughout this step. The following equation can be used to solve for the x-year zero-coupon risk free swap rate of the 3-month zero-coupon risk free curve:

$$100 = SPR_x \cdot \frac{1}{4} \cdot 100\left[\sum_{i=1}^{n} e^{-R_i \times \tau_i}\right] + 100e^{-R_n \times \tau_n} \qquad (3.3)$$

where SPR_x is the x-year swap par rate as quoted in the market, n is the number of 3-month periods in x-years, R_i is the zero-coupon risk free rate for quarter i and τ_i is the time period in terms of day count convention until quarter i.

The procedure for bootstrapping the long end of the zero-coupon risk free swap curve follows as:

i) First the 3-, 6-, 9-, 12-, 15-, 18- and 21-month swap par rates are determined from the calculated zero-coupon risk free rates. For example the 21-month swap par rate is determined by substituting the 3-, 6-, 9-, 12-, 15- and 18-month zero-coupon risk free swap rates, that were determined earlier, into equation (3.3) and solving for the unknown 18-month swap par rate. These swap par rates are determined such that the bootstrap procedure along with the interpolation method delivers more accurate zero-coupon risk free rates for the long end of the zero-coupon risk free swap curve.

ii) The 2-year zero-coupon risk free swap rate is determined. This is done by substituting the 2-year swap par rate as observed in the market along with the prior determined 3-, 6-, 9-, 12-, 15-, 18- and 21-month zero-coupon risk free swap rates into equation (3.3) and solving for the unknown 2-year zero-coupon risk free rate.

iii) The 3-year zero-coupon risk free swap rate cannot be determined as the 27-, 30- and 33-month zero coupon rates are unknown.

iv) First the 27-month zero-coupon risk free swap rate is determined. This is done by using the monotone preserving $r(\tau)\tau$ interpolation to determine a swap par rate for 27-months.

This swap par rate along with the prior determined 3-, 6-, 9-, 12-, 15-, 18-, 21- and 24-month zero-coupon risk free rates are substituted into equation (3.3) and the 27-month zero-coupon risk free rate is solved.

v) The same procedure will be used to determine all the other long end zero-coupon risk free swap rates. Note that only the 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 12-, 15-, 20-, 25 and 30 year zero-coupon risk free swap rates are recorded as knot points.

vi) Now using monotone preserving $r(\tau)\tau$ interpolation on the 18-month, 21-month, 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 12-, 15-, 20-, 25 and 30 year zero-coupon risk free swap rates, each and every point on the zero-coupon risk free swap curve can be determined from 18-months up to 30-years.

3.2 3-MONTH ZERO-COUPON SWAP CURVE CONSTRUCTION USING LINEAR INTERPOLATION

In this section, a stepwise procedure for constructing the 3-month zero-coupon risk free swap curve using linear interpolation is presented. This method is much simpler as the interpolation function is not cubic. Thus, to determine the points in between the knot points of the 3-month zero-coupon swap curve only one knot point above and one knot point below the unknown value is needed. This simplifies the curve construction process.

3.2.1 Step 1 to Step 4:

These steps are the same as for the monotone preserving $r(\tau)\tau$ 3-month zero-coupon swap curve construction. Keep in mind that the 2-month zero-coupon swap rate could be determined by interpolating linearly between the 1-month and 3-month zero-coupon risk free swap curves. The method for interpolating linearly is described in the next step.

3.2.2 Step 5: Interpolating the short end of the curve

The full term structure up to three months of the zero-coupon risk free swap curve can be determined by interpolating. The knot points used as inputs for interpolation are the 1-day, 1-month, 3-month and 4-month zero-coupon risk free swap rates that were determined in the previous steps. By using the interpolation method proposed in Section 2.2.1 all the short end zero-coupon risk free swap rates can be determined up to 4-months of the thirty year term structure.

3.2.3 Step 6 to Step 8:

These steps are the same as for the monotone preserving $r(\tau)\tau$ 3-month zero-coupon swap curve construction except that linear interpolation replaces monotone preserving $r(\tau)\tau$ interpolation.

3.3 3-MONTH ZERO-COUPON SWAP CURVE CONSTRUCTION USING LOG-LINEAR INTERPOLATION

Constructing the 3-month zero-coupon risk free swap curve using log-linear interpolation is trivial following the construction of the 3-month zero-coupon risk free swap curve using linear interpolation. Each step in the construction of the 3-month zero-coupon risk free swap curve is the same as for construction using linear interpolation except that the log-linear interpolation method is now used which has been presented in Section 2.2.2. In the following section swap valuation with these differently constructed curves will be presented.

3.4 SINGLE-CURVE SWAP VALUATION IN THE SOUTH AFRICAN MARKET USING THE DIFFERENTLY CONSTRUCTED ZERO-COUPON SWAP CURVES

In this section, the process of the single-curve approach to swap valuation will be explained. This is followed by a presentation of the values of a single swap when using the differently constructed zero-coupon risk free curves.

3.4.1 Single-curve swap valuation using a zero-coupon risk free swap curve

The single-curve approach to swap valuation uses a single curve to calculate both forward and discount rates that are used in the valuation of the swap. This means that a single curve will be used to determine the floating rate cash flows and the discount rates for each net cash flow.

It is important to note that with the single-curve approach a swap that has payment dates every quarter, is valued using the 3-month zero-coupon risk free swap curve. Whereas a swap with payments every half year is valued using the 6-month zero-coupon risk free swap curve. It is assumed that the rates from the zero-coupon risk free swap curve, which is used to value the swap, are continuously compounded rates.

The following algorithm describes how to value a swap using the single-curve approach:

i) The date the swap is initiated is called the $Startdate$, the maturity date of the swap is called $Matdate$, the fixed rate of the swap is called $FixR$, the nominal principle of the swap is called NP and the time length of the swap in years is called the $Term$, which is equal to the $Matdate - Startdate$.

ii) Determine the theoretical date of each payment, this is the date on which a swap payment would take place if business day conventions were not taken into account. The first theoretical date is initiated as the $Stardate$, with each subsequent theoretical date equal to the payment date ignoring business day conventions.

iii) For each theoretical date, determine the business day on which a payment would take place if the relevant business day count conventions were taken into account.

iv) Determine the days between each business date.

v) Now a fixed cash flow at each payment date of the swap can be determined as $FixedCF_t = NP \times FixR \times \frac{d_t}{t_y}$ where d_t is the number of days between the business date t and the business date of the previous swap payment. The number of days in a year according to the relevant day count conventions is t_y (South African market $t_y = 365$).

vi) For each payment determine the continuously compounded rate ($NACC_t$), that will be used to determine that floating rate, by taking the business date and reading off from the term structure of the zero-coupon risk free swap curve.

vii) Derive the floating rate for time t, which is a forward rate for that payment by using the following formula:

$$FloatR_t = \frac{1}{\tau_t - \tau_{t-1}} \times \{\exp(NACC_t \cdot \tau_t - NACC_{t-1} \cdot \tau_{t-1}) - 1\}$$

where τ_t is the business day on which that payment falls minus the $Startdate$ of the swap measured in years.

viii) Determine each floating rate cash flow as $FloatCF_t = NP \times FloatR_t \times \frac{d_t}{t_y}$ where d_t and t_y have the usual meanings.

ix) Now the net cash flow at each payment date t can be determined as either $NCF_t = FixedCF_t - FloatCF_t$ for the floating rate payer or $NCF_t = FloatCF_t - FixedCF_t$ for the fixed rate payer.

x) Now only future net cash flows, which are net cash flows that occur after the valuation date, will be considered. The present value of each future net cash flow is determined at the valuation date as $PV(NCF_t) = NCF_t \times e^{-NACC_t \times \frac{b_t}{t_y}}$ where b_t is the business day of payment t minus the valuation date.

xi) Taking the sum of these present values will give the value of the swap.

3.4.2 Single swap values at the initiation of each swap

This section will present the different values obtained for a single swap when using differently constructed zero-coupon risk free swap curves for valuation in the South African market. The single-curve swap valuation approach that has been described in the previous sub section will be implemented.

Each swap considered has an initiation date of 30 June 2014, a notional principle of R1million and quarterly payments. Variations, in terms of time to maturity, of the swap have been considered. The 3-month zero-coupon risk free swap curves have been constructed using three different methods of interpolation, namely monotone preserving $r(\tau)\tau$, log linear and linear interpolation.

Table 2: The values of a swap at initiation using differently constructed curves

	Monotone Preserving $r(\tau)\tau$	Linear	Log Linear
2 year	-R 12.54	R 168.59	R 168.59
5 year	-R 41.04	R 364.97	R 364.96
10 year	-R 73.07	R 609.45	R 609.43
15 year	-R 67.50	R 743.86	R 743.84
20 year	-R 31.72	R 686.99	R 686.97
25 year	-R 9.41	R 726.34	R 726.32

Table 2, gives the values of an x-year swap for the floating rate payer at initiation, using the differently constructed 3-month zero-coupon risk free swap curves along with the single-curve approach. The value of a swap at initiation should be zero. Thus, the deviation from zero implies a possible error in the value calculation of the swap from using a wrongly constructed 3-month zero-coupon risk free swap curve.

From the results in Table 2, it can be seen that the error in value of the swap is the smallest for the 3-month zero-coupon risk free swap curve that was constructed using monotone preserving $r(\tau)\tau$ interpolation. From Table 2, it is evident that the error in value is very small for all the 3-month zero-coupon risk free swap curves in comparison to the R1million notional principles of these swaps. This implies that using the differently constructed 3-month zero-coupon risk free swap curves would only lead to a small error in the value of a swap. However, this small error could be magnified for entities that hold a large portfolio of swaps. In the next section, swaps with a variation in both time to maturity and swap par rate will be valued using the differently constructed 3-month zero-coupon risk free swap curves.

3.4.3 Single swap value differences with varying swap par rates and time to maturity

In this section, swaps, with a varying time to maturity and swap par rates, will be valued using the single-curve approach. The differently constructed 3-month zero-coupon risk free swap curves along with the observed market 3-month zero-coupon risk free swap curve will be used to value the swaps. The 3-month zero-coupon risk free swap curves have been constructed using three different methods of interpolation, namely monotone preserving $r(\tau)\tau$, log linear and linear interpolation. The swap values obtained by using the differently constructed swap curves in the valuation will be subtracted from the swap values obtained by using the observed market curve in the valuation. These differences in swap value are seen as a possible error in the value calculation and will be presented accordingly. Each swap considered has a notional principle of R1million, quarterly payments and will be valued on 30 June 2014. Variations, in terms of time to maturity, of the swap have been considered.

Table 3: Possible error in swap value at a swap par rate of 4%

	Swap Par Rate of 4%		
	Monotone Preserving r(t)t	Linear	Log-Linear
5 years	R 25.28	R 381.04	R 379.23
10 years	-R 13.28	R 530.36	R 527.30
15 years	-R 26.76	R 460.02	R 453.99
20 years	-R 14.80	R 115.94	R 111.13
25 years	R 5.28	R 145.69	R 142.25

Table 4: Possible error in swap value at a swap par rate of 6%

	Swap Par Rate of 6%		
	Monotone Preserving r(t)t	Linear	Log-Linear
5 years	R 25.37	R 409.52	R 408.72
10 years	-R 16.33	R 593.42	R 591.80
15 years	-R 32.92	R 597.08	R 593.70
20 years	-R 21.69	R 364.55	R 361.82
25 years	-R 0.61	R 400.91	R 398.97

Table 5: Possible error in swap value at a swap par rate of 8%

	Swap Par Rate of 8%		
	Monotone Preserving r(t)t	Linear	Log-Linear
5 years	R 25.46	R 437.99	R 438.22
10 years	-R 19.38	R 656.47	R 656.30
15 years	-R 39.08	R 734.15	R 733.42
20 years	-R 28.59	R 613.16	R 612.51
25 years	-R 6.50	R 656.14	R 655.70

Table 6: Possible error in swap value at a swap par rate of 10%

	Swap Par Rate of 10%		
	Monotone Preserving r(t)t	Linear	Log-Linear
5 years	R 25.55	R 466.47	R 467.71
10 years	-R 22.43	R 719.53	R 720.80
15 years	-R 45.25	R 871.21	R 873.14
20 years	-R 35.49	R 861.76	R 863.20
25 years	-R 12.39	R 911.37	R 912.43

Table 7: Possible error in swap value at a swap par rate of 12%

	Swap Par Rate of 12%		
	Monotone Preserving r(t)t	Linear	Log-Linear
5 years	R 25.64	R 494.95	R 497.21
10 years	-R 25.48	R 782.58	R 785.30
15 years	-R 51.41	R 1 008.28	R 1 012.85
20 years	-R 42.38	R 1 110.37	R 1 113.88
25 years	-R 18.28	R 1 166.59	R 1 169.15

Tables 3-7, gives the possible error in value calculation for a swap, with x-years left until maturity, when each constructed 3-month zero-coupon risk free swap curve is used in the valuation as opposed to the observed market 3-month zero-coupon risk free curve. In each table a different swap par rate was used to calculate the swap values. These swap values were used to find the possible error in swap value for each swap.

In each of the tables presented, it can be seen that the swap values obtained from using the 3-month zero-coupon risk free swap curve constructed using monotone preserving $r(\tau)\tau$ interpolation produces the smallest possible error in swap value calculation. The 3-month zero-coupon risk free swap curves constructed using linear and log-linear interpolation produce seemingly larger possible errors in the value calculation for the swaps. As larger swap par rates are used in the valuation of the swaps, there is an increase in the possible error of swap value calculation when both of the 3-month zero-coupon risk free swap curves constructed using linear and log-linear interpolation are applied in the valuation. This cannot be said for the 3-month zero-coupon risk free swap curve constructed using monotone preserving $r(\tau)\tau$ interpolation. From each of the tables it is evident that all of the possible errors in swap value calculation are small compared to each swaps notional principle of R1million.

A smaller possible error in swap value calculation implies a smaller error in the construction of the 3-month zero-coupon risk free swap curve compared to the observed market 3-month zero-coupon risk free swap curve. This is because the possible error in swap value calculation is, the swap value obtained from using the observed market 3-month zero-coupon risk free swap curve subtracted from the swap value obtained from using the 3-month zero-coupon risk free swap curve constructed using whichever method of interpolation.

3.5 CONCLUSION

From the results presented in Section 3.4.2 it is evident that the monotone preserving $r(\tau)\tau$ interpolation method applied in the construction of the 3-month zero-coupon risk free swap curve is preferred to the other interpolation methods. Table 2, presents the initial values for a swap when using the differently constructed 3-month zero-coupon risk free swap curves for valuation. The initial swap values obtained when using the 3-month zero-coupon risk free swap curve, that was constructed using monotone preserving $r(\tau)\tau$ interpolation, for valuing each swap are the closest to zero. This implies that the 3-month zero-coupon risk free swap curve that was constructed using monotone preserving $r(\tau)\tau$ interpolation is preferred for valuing swaps. Even though all the values in the table are small in comparison to each swaps notional principle of R1million, this small error is easily magnified when an entity has a large portfolio of swaps. Thus, the smallest error is preferred.

The results in Section 3.4.3 indicate that valuing swaps with the 3-month zero-coupon risk free swap curve that was constructed using monotone preserving $r(\tau)\tau$ interpolation is preferred. This can be seen in Tables 3-7. A possible error in swap value calculation of zero indicates that the swap curve used in the valuation of the swap gives the exact same swap value that would be obtained if the swap was valued using the observed market 3-month zero-coupon risk free swap curve. Thus, a smaller possible error in swap value calculation would indicate that the swap curve that is used in the valuation of the swap, determines swap values as well as the observed market 3-month zero-coupon risk free swap curve. The possible errors in swap value calculation are the smallest for the 3-month zero-coupon risk free swap curve that was constructed using monotone preserving $r(\tau)\tau$ interpolation. Thus, this 3-month zero coupon risk free swap curve is preferred for valuation even though the values in each table is small in comparison to each swaps notional principle of R1million.

Taking into account all the results in this chapter it can be concluded that the 3-month zero-coupon risk free swap curve that is constructed using monotone preserving $r(\tau)\tau$ interpolation would be the preferred constructed swap curve. This is because this swap curve would value swaps with the smallest possible error compared to the swap curves constructed using linear and log-linear interpolation.

CHAPTER 4
SINGLE-CURVE AND MULTI-CURVE SWAP VALUATION IN THE UK MARKET

In this chapter, the single-curve and multi-curve swap valuation approaches are implemented in the UK market. These different swap valuation approaches are implemented to highlight the possible error in value calculation of swaps in markets that still use the single-curve approach for swap valuation. The reason for implementing swap valuation in the UK is because swaps are already being valued using the multi-curve approach in this market and because the UK market has a liquid OIS market (Barbashova, 2012:1).

The multi-curve approach to swap valuation is implemented in the same way as the for the single-curve swap valuation approach described in Section 3.4.1 except that the OIS zero-coupon curve is used for discounting each net cash flow of the swap. In the following section the possible error in values, when using the single-curve swap valuation approach as opposed to the multi-curve swap valuation approach, for different swaps will be presented.

4.1 THE POSSIBLE ERROR IN SWAP VALUE WHEN USING THE SINGLE-CURVE SWAP VALUATION APPROACH

A number of different swaps with a notional principle of £1million are valued on 2014/09/10 using both the single-curve and multi-curve approach. The swaps differ with the amount of payments per year and their amount of time to maturity. The swaps will be valued using different swap par rates. The OIS, 1-, 3- and 6-month zero-coupon risk free swap curves observed in the market on 2014/09/10 were used in the valuation of the swaps. The possible error in swap value is calculated as the swap value obtained from using the single-curve approach subtracted by the swap value obtained from using the multi-curve swap valuation approach.

Table 8: Possible error in swap value, at a swap par rate of 2% for the single-curve approach

	Swap par rate of 2%		
	Payments every month	**Payments every 3-months**	**Payments every 6-months**
5 years	£34.78	£67.24	£170.56
10 years	£543.09	£837.13	£1 746.55
15 years	£1 585.15	£2 335.67	£4 542.32
20 years	£2 778.18	£4 032.94	£7 539.20
25 years	£3 719.28	£5 398.06	£9 898.42

Table 9: Possible error in swap value, at a swap par rate of 2.5% for the single-curve approach

	Swap par rate of 2.5%		
	Payments every month	Payments every 3-months	Payments every 6-months
5 years	-£53.68	-£67.38	-£43.43
10 years	£164.92	£297.30	£864.03
15 years	£792.63	£1 229.03	£2 711.40
20 years	£1 544.15	£2 320.34	£4 706.47
25 years	£2 055.32	£3 089.55	£6 093.26

Table 10: Possible error in swap value, at a swap par rate of 3% for the single-curve approach

	Swap par rate of 3%		
	Payments every month	Payments every 3-months	Payments every 6-months
5 years	-£142.14	-£202.00	-£257.43
10 years	-£213.26	-£242.54	-£18.49
15 years	£0.11	£122.39	£880.48
20 years	£310.11	£607.75	£1 873.73
25 years	£391.35	£781.03	£2 288.11

Table 11: Possible error in swap value, at a swap par rate of 3.5% for the single-curve approach

	Swap par rate of 3.5%		
	Payments every month	Payments every 3-months	Payments every 6-months
5 years	-£230.60	-£336.62	-£471.42
10 years	-£591.43	-£782.37	-£901.01
15 years	-£792.41	-£984.25	-£950.44
20 years	-£923.92	-£1 104.85	-£959.00
25 years	-£1 272.61	-£1 527.48	-£1 517.05

Table 12: Possible error in swap value, at a swap par rate of 4% for the single-curve approach

	Swap par rate of 4%		
	Payments every month	Payments every 3-months	Payments every 6-months
5 years	-£319.05	-£471.24	-£685.41
10 years	-£969.60	-£1 322.20	-£1 783.53
15 years	-£1 584.93	-£2 090.89	-£2 781.36
20 years	-£2 157.96	-£2 817.45	-£3 791.73
25 years	-£2 936.57	-£3 836.00	-£5 322.21

Tables 8-12, present the possible errors in the value of an x-year swap, with varying payment frequencies, when the single-curve approach is used for swap valuation. In each table, a different swap par rate has been used to calculate both the single-curve and multi-curve swap value for each swap. These swap values were then subtracted to determine the possible error in value for each swap.

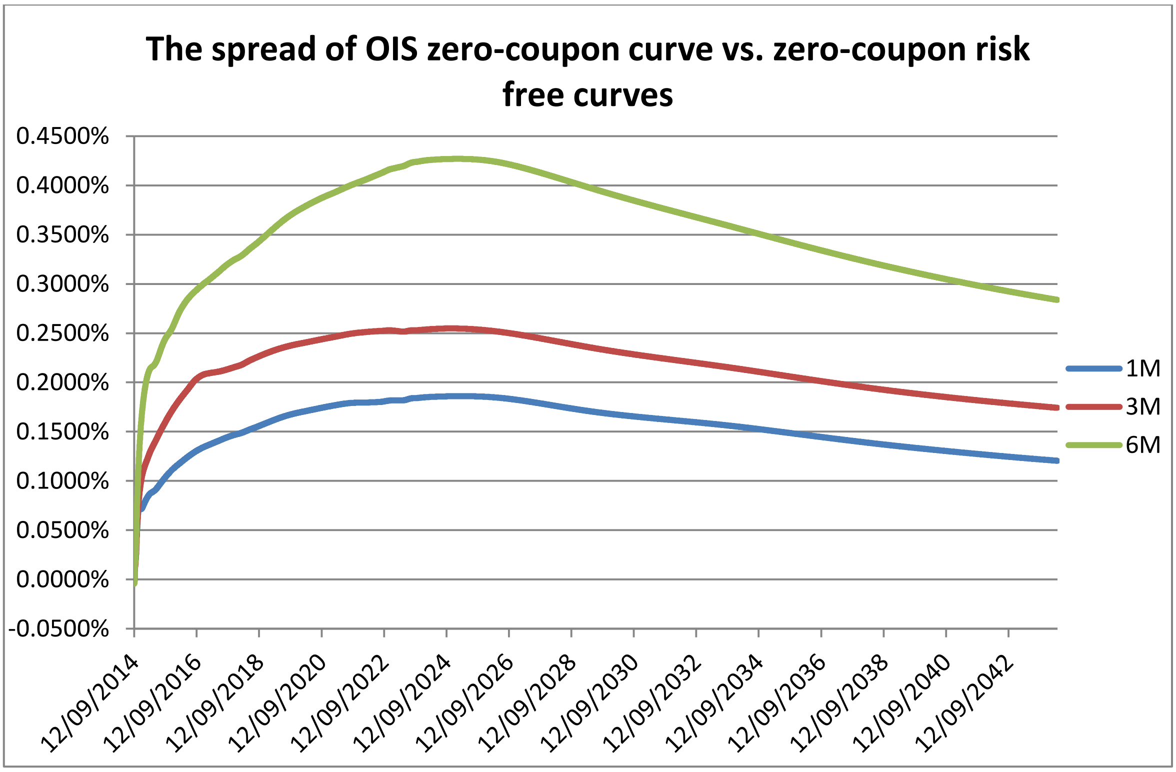

Figure 3: Spread between OIS zero-coupon curve vs. 1-, 3- and 6-month zero-coupon risk free swap curve

Firstly, the possible errors in swap values are more significant for a swap that has a smaller amount of payments per year. This can be explained by looking at the spread between the OIS zero-coupon curve and each of the zero-coupon risk free swap curves as presented in Figure 6. In Figure 6, 1M represents the spread between the OIS zero-coupon curve and the 1-month zero-coupon risk free swap curve, 3M represents the spread between the OIS zero-coupon curve and the 3-month zero-coupon risk free swap curve and 6M represents the spread between the OIS zero-coupon curve and the 6-month zero-coupon risk free swap curve.

The spread between the OIS zero-coupon curve and the 6-month zero-coupon risk free swap curve is larger than for the 1- and 3-month zero coupon risk free swap curves. This is because the rates used for constructing the 6-month zero-coupon risk free swap curve contain credit risk inherent in a loan for the duration of 6-months, which would be larger than the credit risk inherent in a loan for 1- or 3-months. The credit risk inherent in a loan for 3-months is larger than the credit risk inherent in a loan for 1-month. The credit risk inherent in the inputs for the OIS zero-coupon curve is negligible. Thus, the spread between the OIS zero-coupon curve and the 1-month zero-coupon risk free swap curve would be the smallest as the amount of credit risk inherent in one month is smallest compared to 3- or 6-months.

Finally, from Table 8-12 it can be seen that as the time to maturity of a swap increase, the possible error in value calculation become larger when using the single-curve approach to swap valuation. For example, in Table 12 the 5-year swap with payments every three months has a possible error in value of -£471.24 whereas the value difference for the 15-year swap with payments every three months at that time is -£2090.89. The value difference for the 25-year swap with payments every three months at that time is -£3836.00. Thus, the difference in the values obtained for a swap when using the two different approaches to swap valuation will be much larger as the time to maturity of a swap increases.

This is explained by the usage of the single-curve and multi-curve swap valuation approaches. The multi-curve swap valuation approach uses the OIS zero-coupon curve for discounting the future net cash flows of a swap. Whereas the single-curve swap valuation approach uses the same zero-coupon risk free swap curve for both calculating forward rates and discounting the future net cash flows of the swap. As the lifetime of a swap increases, it means that there is an increase in the number of future net cash flows of the swap that need to be discounted to determine its value. As the number of future net cash flows increase, it implies that more net cash flows need to be discounted to find the value of the swap. Now because a curve other than the OIS zero-coupon curve is used for discounting there is a possible error in the present value of each future net cash flow. Thus, the possible error in the value of a swap is increased as the value of a swap is the sum of the discounted future net cash flows.

4.2 CONCLUSION

The results in this chapter indicate that there is a possible error in swap value calculation from using the single-curve swap valuation approach as opposed to the multi-curve swap valuation approach. It can be seen that for a swap with a longer maturity the possible error in value calculation is larger than for a swap with a shorter maturity. The payment frequency of a swap would also have an impact on the possible error in swap value calculation depending on the size of the spread between the x-month zero-coupon risk free swap curve and the OIS zero-coupon risk free curve. Even if each possible error in swap value is small in comparison to its notional principle of £1million, this possible error could be magnified for entities that hold a large portfolio of swaps.

CHAPTER 5
CONCLUSION

Risk neutral swap valuation is of importance as it is the basis value on which non-performance risks are added to determine the market value of a swap. The zero-coupon risk free swap curve is used in determining the risk neutral swap value. In both the single-curve and the multi-curve swap valuation approaches it is important to know how the zero-coupon risk free swap curve is constructed in the market. Also it is important to know what the possible error in the risk neutral swap value would be if the single-curve approach to swap valuation were implemented as opposed to the multi-curve swap valuation approach. This is because the single-curve swap valuation approach is still being implemented in some undeveloped markets as opposed to the relatively new multi-curve approach that is implemented in most developed markets.

The construction of the 3-month zero-coupon risk free swap curve in the South African market was presented in Chapter 3. This swap curve was constructed using three different methods of interpolation namely monotone preserving $r(\tau)\tau$, linear and log-linear interpolation. It was found that constructing the 3-month zero-coupon risk free swap curve using monotone preserving $r(\tau)\tau$ interpolation would give the most optimal curve in comparison to the observed market curve.

In Chapter 4, the possible error in swap value when using the single-curve approach as opposed to the multi-curve approach for swap valuation is presented. These possible errors were calculated in the UK market. It was found that the possible errors in swap value calculation were small in comparison to the each swaps notional principle. However, for entities that hold a large portfolio of swaps this error in value could be magnified. These results indicate that markets which still use the single-curve approach to swap valuation instead of the newer multi-curve swap valuation approach, could possibly value swaps incorrectly. As an entity holds a large portfolio of swaps the error in portfolio value could be large as each swap could be valued incorrectly.

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