Estimating Expected Exposure for Counterparty Credit Risk Adjustments

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Abstract

Counterparty credit risk (CCR) forms an integral part of the current financial market. CCR is already widely implemented by banks under the Basel Committee of Banking Supervision guidelines. The International Accounting Standards Board has introduced IFRS 13, which subsequently requires all financial entities to use fair value adjustments to account for CCR. In this paper the methods to estimate expected exposure (EE) of interest rate swaps for credit valuation adjustments (CVA) are discussed and compared to attempt to find the most appropriate method to be adopted by smaller non-financial organisations.

A model is developed to estimate EE with the Monte Carlo methodology using the Vasicek short rate model. The more computationally intensive Monte Carlo method is then compared to add-on methodologies such as the current exposure method (CEM) and expansions thereof. Thereafter, the CVA values are calculated using each method and compared. Finally, the results of each method were presented and then discussed in terms of accuracy and simplicity.

It was found that when comparing the EE estimated with the simple CEM method to the Monte Carlo method, the results were inaccurate. However, by making simple modifications to the simple CEM method, the accuracy of estimation can be improved greatly. Therefore, although the Monte Carlo method is still more accurate, smaller non-financial organisations with resource constraints and fewer technical abilities could use the modified CEM method an approximation for CVA.

Key words:

CCR; CVA; Expected Exposure; CEM; Monte Carlo; Vasicek; Interest Rate Swap; IFRS

Opsomming

Teenparty-kredietrisiko vorm ‘n integrale deel van die finansiële mark en word reeds deur banke geïmplementeer om aan die Basel-Komittee vir Banktoesighouding riglyne te voldoen. Die Internasionale Rekeningkundige Standaarde Raad het IFRS 13 ingestel, wat gevolglik vereis dat alle finansieële instansies billikewaarde-aanpassings gebruik om vir CCR voorsiening te maak. Die metodes om die verwagte blootstelling van rentekoersruiltransaksies, vir krediewaarde-aanpassings, te beraam word in hierdie navorsingstuk bespreek en vergelyk om moontlik die mees toepaslike metode te vind wat deur kleiner nie-finansiële organisasies gebruik kan word.

Die Vasicek model is gebruik om ‘n Monte Carlo model te ontwikkel om die verwagte blootstelling te beraam. Hierdie model wat meer intesief in terme van berekening is word vergelyk met ander metodes soos die Huidige Blootstellings Metode en uitbreidings van die metode. Die waardes van die kredietwaarde-aanpassings word daarna bereken. Die resultate van elke metode word voorgestel en bespreek in terme van eenvoud en akkuraatheid.

Daar is gevind dat wanneer die gewone huidige blootstellings metode met die Monte Carlo metode vergelyk word, is die resultate onakkuraat. Daar kan egter eenvoudige aanpassings aan die Huidige Blootstellings Metode gemaak word wat die akkraatheid aansienlik verbeter. Dus, kan die aangepaste Huidige Blootstellings Metode deur kleiner nie-finansiële organisasies met beperkte hulpbronne gebruik word.

Sleutelwoorde:

Teenparty-kredietrisiko; Verwagte Blootstelling; Monte Carlo; Rentekoersruiltransaksie; IFRS

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List of abbreviations and/or acronyms

CCR	Counterparty Credit Risk
CVA	Credit Valuation Adjustment
DVA	Debit Valuation Adjustment
EE	Expected Exposure
MtM	Mark-to-Market
IASB	International Accounting Standards Board
OTC	Over-the-Counter
PD	Probability of Default
LGD	Loss Given Default
IRS	Interest Rate Swap
JIBAR	Johannesburg Inter Bank Agreed Rate
LIBOR	London Inter Bank Offer Rate
PFE	Potential Future Exposure
VaR	Value-at-Risk
CEM	Current Exposure Method
EAD	Exposure at Default
CCF	Credit Conversion Factor
PV	Present Value

CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

The concept of default and its financial repercussions have been well established in history and understood by investors. There have been many examples, including Sovereign entities such as Russia (1998) and Argentina (2001), and Corporates such as Long Term Capital Management (1998), WorldCom Inc. (2002), and Lehman Brothers (2008). These types of default events and the subsequent financial implications lead to the development of credit risk management (FinancialCAD Corporation, 2011:2).

One of the primary focus points of Basel III rules on minimum capital requirements issued by the Basel Committee of Banking Supervision has been counterparty credit risk (CCR). CCR is defined as the risk that the counterparty will fail to fulfil an obligation in the future (Deloitte, 2013:1). The effort to quantify CCR has resulted in a divergence between accounting and regulatory standards (Shearman & Sterling, 2013:2). As a result, the International Accounting Standards Board issued IFRS 13 *Fair Value Measurement* to reduce the inconsistencies applied in practice and to unite international and US accounting treatments (Gregory, 2012:272). IFRS 13 has implemented the use of fair value adjustments for the risk of counterparty default as well as for the institutions own non-performance risk. These adjustments, which account for CCR, are referred to as credit valuation adjustment (CVA) and debit valuation adjustment (DVA). CVA is essentially an adjustment to the measurement of derivative assets to reflect the default risk of the counterparty whereas DVA is an adjustment to the measurement of derivative liabilities to reflect the own default risk of the entity (Kengla and De Jonghe, 2013:4).

Many banks have already accounted for CCR in their annual financial statements (AFS) but the recent global financial crisis has forced a more active and accurate approach and thus dynamically pricing CCR directly into new trades (Algorithmics, 2009:2). While CVA pricing methodologies are well advanced, they are still not standardised and may vary amongst market participants, ranging from relatively simple to highly complex methodologies that are driven largely by the sophistication and resources available to the market participant (Gregory, 2012:157). Depending on the particular market participant, CVA can be significant, particularly for large financial institutions that are highly active in the derivative markets (FinancialCAD Corporation, 2011:3).

This study aims to standardise the methodology used when quantifying expected exposure (EE) for CVA for market participants such as smaller non-financial organisations that cannot afford to implement advanced models. Further, considering the definition of DVA, the assumption will be made that the method for calculating DVA is similar to CVA and therefore this study will only focus on CVA.

1.2 PROBLEM STATEMENT

In order to determine CCR fair value adjustments, institutions need to quantify and measure the exposure that could be lost if the counterparty defaults. Thus, the EE needs to be estimated. In broad terms, EE is the exposure to the counterparty at the point of default – this could be either positive or negative (Gregory, 2012:127). There are numerous techniques for calculating EE for CVA and determining which EE methodology to implement is often driven by the institution's sophistication, technical ability and resource constraints. These approaches range from simplistic to advanced, while the level of accuracy is mostly dependent on the method. Determining a point where the compromise of accuracy can be justified by the level of simplicity will be beneficial to the market participants and will contribute to standardising the methodology for estimating CVA across the market.

1.3 RESEARCH OBJECTIVES

The aim of this research assignment is to attempt to investigate the most efficient method for calculating EE in terms of the level of sophistication, accuracy and the operational considerations. By applying a the more simplistic methodology such as current mark-to-market (MtM) plus an add-on to an interest rate swap between an institution and counterparty, it can be compared to the more complex Monte Carlo approach to quantifying EE. The effects of netting and collateral agreements on EE will also be discussed briefly. The methods will be compared in order to recommend the one with the optimal trade off between the levels of simplicity versus the level of accuracy when estimating EE.

1.4 IMPORTANCE/BENEFITS OF THE STUDY

IFRS 13 issued by the International Accounting Standards Board (IASB) have greatly emphasised CCR. As a result, all financial entities will need to comply with the new CCR regulations. These changes have been widely accepted in Europe, but are yet to be commonly implemented in South Africa by smaller non-financial organisations and in other developing countries due to the numerous methodologies available, operational constraints and insufficient resources of smaller entities. CCR has rapidly become the problem of all financial institutions, big or small, since there has been an increase in hedging and a tightening of traditional risk mitigation methods (Algorithmics, 2009:5). The rapid growth in the over-the-counter (OTC) derivatives market has been reversed, at least temporarily, which emphasizes the need for better CCR management, as this will allow trading activity to increase while reducing the chance of significant future losses or systemically driven market disturbances (Algorithmics, 2009:5).

1.5 RESEARCH DESIGN AND METHODOLOGY

The research assignment will be based on four parts. The first part will consist of an in depth literature review covering the necessary theoretical knowledge required on estimating EE. The second part will focus on the design and building of a programme in a statistical software package to price a plain vanilla interest rate swap. To price the swap, the underlying variables such as the Johannesburg Inter Bank Agreed Rate (JIBAR) interest rate, the swap rate, maturity, payment frequency and the notional amount will be used. The third part of the research assignment will entail applying each method to estimate EE, including the Monte Carlo method, considered in the literature review. To estimate EE with the Monte Carlo method, a programme will also be written in a statistical software package to simulate the value of the swap at the payment dates in the future by simulating the swap curve.

The time horizon until the point of default, which will be assumed in all estimations of EE, will be varied to compare the different methods for short-term and long-term interest rate swaps. This research assignment will focus on estimating the EE and thus the probability of default (PD) of the counterparty, as well as the loss given default (LGD), will be assumed as an arbitrary value. This assumption will be consistent in all calculations of CVA. Consequently, the PD and LGD can be stressed to examine the effect on the value of CVA. Finally, the data output from each method will be compared to determine the most appropriate method for estimating EE.

1.6 CONCLUSION

This chapter gave an introduction and background, as well as the problem statement to for this study. The next chapter focuses on the review of literature for the estimation of exposure. In the chapter thereafter, a discussion on the specific methodology employed in the study will be provided. The results of the different methodologies for estimating exposure as well as the comparison will be discussed in the penultimate chapter and then concluded in the final chapter.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

Credit valuation adjustment (CVA) is an adjustment to the measurement of derivative assets to reflect the default risk of the counterparty. Quantifying CVA is relatively simple, consisting of the combination of the probability of default (PD), the loss given default (LGD) and exposure.

According to Gregory (2012:157), a balance between the following two effects lies at the heart of quantifying exposure:

- When observing the future, the market variables become increasingly uncertain. Hence, risk increases when moving through time.
- Many financial instruments have cash flows that are paid over time and this tends to reduce risk profiles as the instruments amortise through time.

In the first section basic definitions that are necessary to understanding CVA will be given. Thereafter the formal definition of CVA will be discussed followed by discussions on PD and LGD. In the sections that follow, the various methods to quantify exposure such as mark-to-market (MtM) value plus add-on and Monte Carlo methodology will be elaborated on.

2.2 BASIC DEFINITIONS

2.2.1 The Money Market Account

Let B_t be the value of 1 unit of money at time t, invested in the account at time zero. The money market account is assumed to evolve under the following differential equation

$$dB_t = r_t B_t dt, \qquad (2.1)$$

where r_t is the instantaneous interest rate at time t, known as the short rate. Therefore, the value at time t of the money market account at time t is

$$B_t = \exp\left(\int_0^t r_s ds\right) \qquad (2.2)$$

(Brigo and Mercurio, 2006:2; Jones, 2010:11).

2.2.2 Zero Rates

The t-year zero-coupon interest rate, $R(0,t)$, is the rate of interest earned on an investment that starts today and lasts for t years. All the interest and principal is realised at the end of the t years (Hull, 2009:78). Formally, the continuously compounded zero rate at time t for maturity T, denoted $R(t,T)$, is the constant which rate of interest an investment of $P(t,T)$ at time t would need to earn to yield 1 unit of money at time T, i.e.

$$P(t,T) = 1 \cdot e^{-R(t,T)(T-t)}$$
$$\Rightarrow R(t,T) = -\frac{1}{T-t}\ln(P(t,T)) \tag{2.3}$$

(Jones, 2010:11).

2.2.3 Forward Rates

Forward rates are interest rates implied by the current zero rates for certain periods of time in the future and denoted by $F(T_{i-\delta},T_i)$ (Hull, 2009:82) and calculated as

$$F(T_{i-\delta},T_i) = \frac{R(0,T_i)T_i - R(0,T_{i-\delta})T_{i-\delta}}{T_i - T_{i-\delta}}, \tag{2.4}$$

where δ denotes one period of time.

2.2.4 Definition of an Interest Rate Swap

An interest rate swap (IRS) is an agreement between two parties to exchange cash flows in the future. The agreement defines the dates the cash flows are to be paid on and the way the payments are to be calculated. A plain vanilla IRS is when a party agrees to pay a cash flow equal to a predetermined fixed rate on a notional principal for a number of years. The same party in return receives interest at a floating rate on the same notional principal for the same period (Hull, 2009:147).

According to Jones (2010:14), in the South African market the fixed and floating legs are typically exchanged quarterly. Figure 2.1 below illustrates a plain vanilla IRS where party A pays the fixed rate (5%) and receives the floating JIBAR plus a spread, whilst party B pays the floating JIBAR plus a spread and receives the fixed rate.

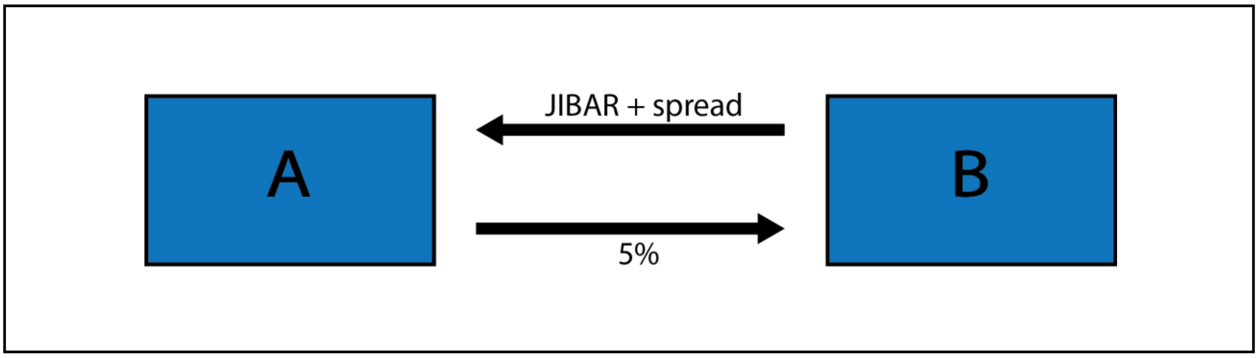

Figure 2.1: Illustration of a plain vanilla IRS

2.2.5 LIBOR and JIBAR

The London Inter Bank Bid-Offer Rate (LIBOR) is defined as the rate at which a bank is willing to lend to other banks in London (Hull, 2009:147). This rate is published daily for a range of borrowing periods. JIBAR is the South African equivalent of LIBOR.

2.2.6 Risk-neutral Valuation

In a risk neutral world all individuals are indifferent to risk and do not need compensation for risk. Hence, the expected return on all investments is just the risk-free rate (Van der Merwe, 2010:76). It is therefore true that the present value of any cash flow in a risk neutral world can be obtained by discounting its expected value with the risk-free interest rate (Hull, 2009:290).

A derivative with a payoff at a particular time can then be valued using risk-neutral valuation by using the following procedure:

1. Assume that the expected return on the underlying asset is the risk-free interest rate.
2. Calculate the expected payoff of the derivative.
3. Discount the expected payoff at the risk-free interest rate.

Risk-neutral valuation and the expectation with respect to the risk-neutral world will be formally presented in Appendix A. When there is a move from the risk-neutral world to a risk-averse world, the expected growth of a stock price changes and the discount rate to be used for the derivative payments changes. These changes offset each other exactly, implying that that the risk-neutral valuation assumption is valid in all worlds. Therefore, risk-neutral valuation is appropriate to use for the simulation (Hull, 2009:290).

2.3 CREDIT VALUATION ADJUSTMENT

Traditionally, the price of a derivative has been calculated using risk-neutral pricing. This yields the fair value that has been offered to other market participants without adjustments to account for counterparty credit risk (CCR) (Ahlberg, 2013:7; Gregory, 2012:241). With the introduction of CVA, the CCR can therefore be incorporated in the fair valuation of financial assets.

2.3.1 Definition Of Credit Valuation Adjustment

The CVA of a derivative is defined as the difference between the risk-free value, when assuming no CCR, and the risky value defined as,

$$\tilde{V}(t,T) = V(t,T) - CVA(t,T), \tag{2.5}$$

where $V(t,T)$ denotes the risk-free fair value of the derivative contract at time t with a maturity date at time T.

To derive Equation 2.5, denote the default time of the counterparty as τ and $V(s,T), t \leq s \leq T$ as the future uncertain fair value at time s, accounting for the effects of discounting. Firstly, consider the case where the counterparty does not default before time T, where the risky position is equivalent to the risk-free position and write the payoff as

$$I(\tau > T)V(t,T), \tag{2.6}$$

where $I(\tau > T)$ is the indicator function denoting default. Secondly, consider the case where the counterparty defaults before time T, where the payoff consists of two terms, the value of the position that would be paid before the default time, i.e.

$$I(\tau \leq T)V(t,\tau), \tag{2.7}$$

plus the payoff at the time of default. If the fair value of the trade at the time of default, $V(\tau,T)$, is positive then the institution will receive a recovery fraction, Θ, of $V(\tau,T)$. If $V(\tau,T)$ is negative, the amount will still need to be settled. Hence, the payoff at the time of default, τ, is

$$I(\tau \leq T)(\Theta V(\tau,T)^{+} + V(\tau,T)^{-}), \tag{2.8}$$

where $x^{-} = \min(\mathrm{x},0)$ and $y^{+} = \max(y,0)$.

From Equations 2.6, 2.7 and 2.8, it follows that the value of the risky position is

$$
\begin{aligned}
\tilde{V}(t,T) &= E^{\mathbb{Q}}\begin{bmatrix} I(\tau > T)V(t,T) + \\ I(\tau \leq T)V(t,\tau) + \\ I(\tau \leq T)(\Theta V(\tau,T)^{+} + V(\tau,T)^{-}) \end{bmatrix} \\
&= E^{\mathbb{Q}}\begin{bmatrix} I(\tau > T)V(t,T) + \\ I(\tau \leq T)V(t,\tau) + \\ I(\tau \leq T)(\Theta V(\tau,T)^{+} + V(\tau,T) - V(\tau,T)^{+}) \end{bmatrix}, \qquad x^{-} = x - x^{+} \\
&= E^{\mathbb{Q}}\begin{bmatrix} I(\tau > T)V(t,T) + \\ I(\tau \leq T)V(t,\tau) + \\ I(\tau \leq T)\big((\Theta - 1)V(\tau,T)^{+} + V(\tau,T)\big) \end{bmatrix} \\
&= E^{\mathbb{Q}}\begin{bmatrix} I(\tau > T)V(t,T) + \\ I(\tau \leq T)V(t,T) + \\ I(\tau \leq T)((\Theta - 1)V(\tau,T)^{+}) \end{bmatrix},
\end{aligned}
\tag{2.9}
$$

where $E^{\mathbb{Q}}$ is the expected value is with respect to the risk-neutral world. Finally, since $I(\tau > T)V(t,T) + I(\tau \leq T)V(t,T) \equiv V(t,T)$, the risky value can be expressed as

$$
\begin{aligned}
\tilde{V}(t,T) &= V(t,T) - E^{\mathbb{Q}}[(1-\Theta)I(\tau \leq T)V(\tau,T)^{+}] \\
&= V(t,T) - CVA(t,T)
\end{aligned}
$$

The equation for CVA is in fact more complex than it seems, as it is not linear (Gregory, 2012:243). Due to netting and collateral, which will be discussed later, CVA is not an additive quantity with respect to individual transactions. Therefore, $\tilde{V}(t,T)$ cannot be calculated individually, since it is defined with respect to other transactions in the same netting set (Gregory, 2012:12A).

2.3.2 Formula for Credit Valuation Adjustment

Under the above assumptions, a standard equation for CVA can be derived. Consider again the formula for CVA, which can be written as follows

$$CVA(t,T) = (1-\Theta)E^{\mathbb{Q}}[I(u \leq T)V^{*}(u,T)^{+}], \tag{2.10}$$

where Θ is the expected recovery rate and $V^{*}(u,T)^{+}$ denotes

$$V^{*}(u,T) = V(u,T)|\tau = u$$

This is critical in the understanding of exposure estimation, as the above statement requires the exposure at a future date given that the counterparty has defaulted at that particular future date. In this study $V^{*}(u,T) = V(u,T)$, since the effects of wrong-way risk will be ignored, because interest rate products have limited wrong-way risk (Gregory, 2012:309). Wrong-way risk is the term used for the dependence between exposure and counterparty credit quality. When exposure is high, the counterparty is more likely to default (Pykhtin and Zhu, 2007:38).

It is now possible to integrate over all possible default times to obtain

$$CVA(t,T) = (1-\Theta)E^{\mathbb{Q}}\left[\int_t^T DF(t,u)V(u,T)^+ dP_D(t,u)\right], \tag{2.11}$$

where $DF(t,u)$ is the risk-free discount factor and $P_D(t,u)$ is the cumulative default probability for the counterparty. Recognise that the discounted EE is calculated under risk-neutral measure denoted by $EE_d(t,T) = E^{\mathbb{Q}}[DF(t,u)V(u,T)^+]$ (Pykhtin and Zhu, 2007:38). If it is assumed that all probabilities are deterministic, the default time is independent of $DF(t,u)$ and therefore eliminates the need to simulate the probabilities of default. The formula can then be written as

$$CVA(t,T) = (1-\Theta)\left[\int_t^T EE_d(u,T) dP_D(t,u)\right] \tag{2.12}$$

The above equation can then be approximated by Monte Carlo integration (Rizzo, 2008:120), such as

$$CVA(t,T) \approx (1-\Theta)\sum_{i=1}^{N} EE_d(t,t_i)[P_D(t,t_i) - P_D(t,t_{i-1})], \tag{2.13}$$

where N is the number of periods given by $i = t_0, t_1, \ldots, t_N$. To work with the non-discounted expected exposure at the point of default, simply write

$$\begin{aligned} CVA(t,T) &= (1-\Theta)\left[\int_t^T EE(u,T)DF(t,u)dP_D(t,u)\right] \\ &\approx (1-\Theta)\sum_{i=1}^{N} DF_{t_i}EE_{t_i}PD(t_{i-1},t_i), \end{aligned} \tag{2.14}$$

where $(1-\Theta) = LGD$, DF is the relevant risk-free discount factor, EE is the expected exposure on the relevant dates and PD is the probability of default between specified dates (Gregory, 2012:243; Pykhtin and Zhu, 2007:22; Ahlberg, 2013:21).

2.3.3 Summary

CVA therefore depends on combining these three components from potentially different sources. It is crucial to understand that in the derivation of Equation 2.14, the components are assumed to be independent (Gregory, 2012:244). In other words, there can be separate departments responsible for the EE, LGD and PD where none of these departments need to be aware of the other.

Equation 2.14 also provides a further advantage to the calculation of CVA, which is that counterparty default only enters the equation through default probability. This significantly saves on computational time, as it is not necessary to simulate the default events, only the EE (Gregory, 2012:244). In the subsequent sections, the LGD and PD will be discussed briefly, followed by an in depth discussion on the various methods to quantify exposure.

2.4 PROBABILITY OF DEFAULT

The probability of default (PD) is the likelihood that the counterparty will default during a specific period of time or interval. In a financial sense, the counterparty defaults when it is no longer able to fulfil its debt obligations (Norman and Chen, 2013:32).

To define the PD mathematically, define the cumulative default probability function, $P_D(t)$, as the probability of default at any point prior to time t (Ahlberg, 2013:9). The marginal default probability, the probability of default between two specified future dates, is given by

$$q(t_1, t_2) = P_D(t_2) - P_D(t_1), \qquad (t_1 < t_2), \tag{2.15}$$

which is illustrated graphically in Figure 2.2. Note that, $P_D(.)$ for a counterparty which has not yet defaulted must be monotonically increasing with $P_D(0) = 0$ and $\lim_{t\to\infty} P_D(t) = 1$ to avoid negative marginal default probabilities (Ahlberg, 2013:9; Gregory, 2012:198).

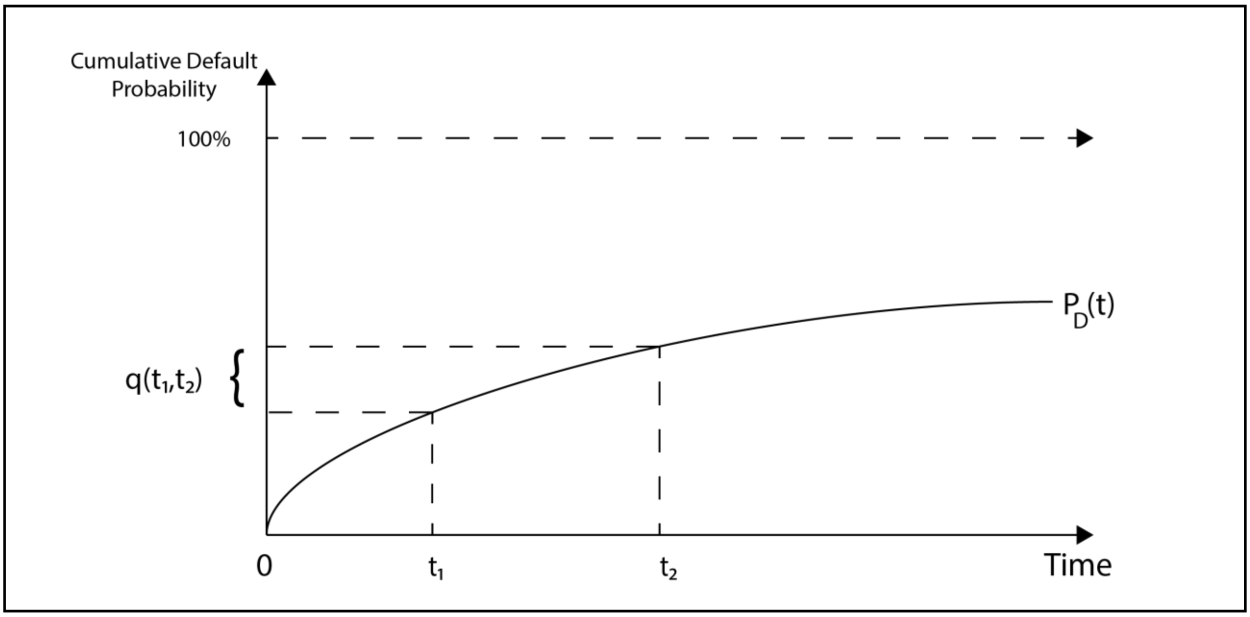

Figure 2.2: The Marginal Default Probability

Source: Gregory, 2012:198

The probability of default can be defined as real world and risk-neutral default probabilities. Real-world default probabilities are estimated from historical data, whilst risk-neutral default probabilities are reflecting parameters derived from market prices. According to Hull (2012:529-530), the question regarding which default probabilities to use, depends on the purpose of the analysis. Risk-neutral default probabilities should be used when valuing credit derivatives or estimating the impact of default risk. When calculating potential future losses by carrying out scenario analysis, real-world default probabilities should be used. Therefore, risk-neutral default probabilities should be used when calculating CVA.

2.5 RECOVERY RATES AND LOSS GIVEN DEFAULT

In the case where the counterparty defaults, there will generally be a fraction of the outstanding claim recovered. This is known as the recovery rate, denoted by Θ. Recovery rates can also be expressed as the loss given default or LGD, defined (Ahlberg, 2013:11) as

$$LGD = 1 - \Theta \tag{2.16}$$

The recovery rate is essentially the ratio of the exposure that would be recovered in the event of default and therefore LGD is the ratio of the exposure that would be lost.

Recovery rates are an important component in the calculation of CVA, but it can be difficult to estimate precisely (Norman and Chen, 2013:31). To estimate recovery rates, the norm is to look to historical analysis and like default probabilities, the values show significant variation over time. Recovery rates also tend to be negatively correlated with default rates, which means a high default rate will give rise to a lower recovery value. This negative correlation coupled with the random nature of default probability and recovery rates creates strong variability in default losses (Gregory, 2012:209-210).

2.6 DEFINITION OF CREDIT EXPOSURE

Credit exposure is simply defined as the loss in the event of the counterparty defaulting (Gregory, 2012:121). Exposure is characterised by the fact that a positive value of a financial instrument corresponds to a claim on a defaulted counterparty, whereas a negative value, an institution is still obliged to honour the contractual agreement (Norman and Chen, 2013:30-31). Hence, an asymmetry of potential losses arises. It is very important to note that exposure is *conditional on counterparty default.* Exposure at time t, conditional on counterparty default, can simply be defined (Pykhtin and Zhu, 2007:17) as

$$Exposure(t) = \max(V(t,T), 0), \tag{2.17}$$

where $V(t,T)$ is the fair value of the contract at time t. Since the value of the contract is a risk-neutral expectation, the exposure is also a risk-neutral expectation (Ahlberg, 2013:8).

There are methods to define exposure more specifically that are given in the Basel Committee of Banking Supervision guidelines, which will be discussed in the following sections. Thereafter, a discussion on the different methods for quantifying credit exposure will be represented.

2.6.1 Expected exposure

The pricing of counterparty credit risk will involve EE that is defined as the average of all the possible exposure values considering the different scenarios depicted in Figure 2.3. Mathematically (Pykhtin and Zhu, 2007:17; Norman and Chen, 2013:37), EE at time t can be calculated as

$$EE_t = \frac{1}{N}\sum_{i=1}^{N} \max\left(V_{t_i}, 0\right), \tag{2.18}$$

where N is the number of generated scenarios and V_{t_i} is the fair value at time t for scenario i. Note that only positive values give rise to exposure whereas negative values have zero contribution.

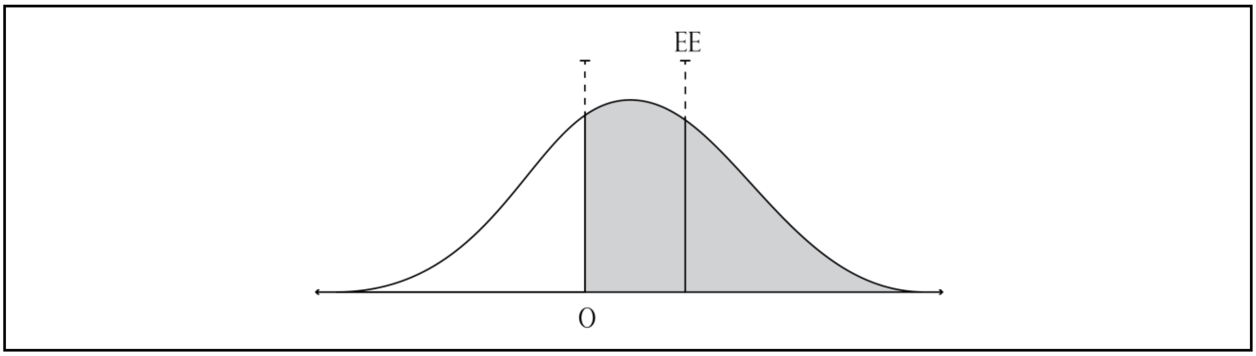

Figure 2.3: Expected Exposure

Source: Gregory, 2012:128

2.6.2 Potential Future Exposure

Potential future exposure (PFE) is an extreme measure of exposure and can be defined as the exact same measure as Value-at-Risk (VaR), with the only differences being that PFE is defined for more than one future date and represents gains instead of losses. VaR will not be discussed in this study, but the reader can see Alexander (2008) for further reading. With PFE it is possible to calculate the worst exposure with reference to a confidence level, which is a natural question to ask in risk management. (Norman and Chen, 2013:31). A PFE confidence level of 99% will define the exposure that would be exceeded with a probability of no more than 1%. PFE is illustrated in Figure 2.4 below. PFE only represents the worst exposure values, while EE represents the average of all values (Gregory, 2012:127).

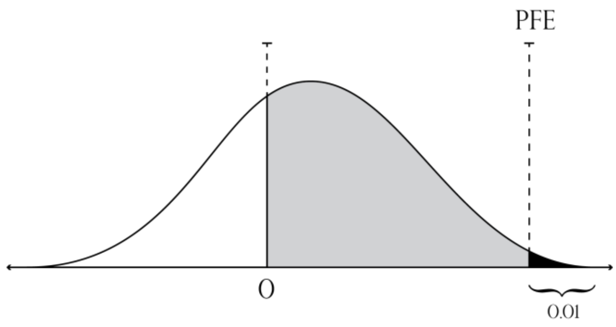

Figure 2.4: Potential Future Exposure

Source: Gregory, 2012:127

2.7 QUANTIFYING CREDIT EXPOSURE

The practical calculation of exposure involves choosing a balance between sophistication and operational considerations (Gregory, 2012:157). In the first section, the MtM plus add-on method will be discussed followed by an in depth discussion on the Monte Carlo methodology.

2.7.1 Mark-to-Market Plus Add-on (Current Exposure Method)

The simplest approach to approximate future exposure is to take the current positive exposure and add a component that represents the uncertainty of the PFE in the future. This approach is known as the current exposure method (CEM). At trade level, an add-on component should always account for the time horizon in question and the volatility of the underlying asset class (Gregory, 2012:157-158).

CEM is the approach that was first introduced in the Basel Accords and is formally defined as equal to the replacement cost of the fair value contract currently in the money, plus the credit exposure risk of potential future volatility of the underlying asset. The CEM is used to calculate the exposure at default (EAD), define as

$$EAD = \max(V_0, 0) + Not \cdot CCF, \tag{2.19}$$

where V_0 is the current MtM value, Not is the notional amount and CCF is the Credit Conversion Factor (Douglas and Pugachevsky, 2012:3-4; Kotzé, 2012:4-5). These factors are fixed and are specified in Basel II given in Table 2.1 on the next page.

Table 2.1: Conversion Factor Matrix for OTC Derivative Contracts

	Type						
Remaining Maturity (Years)	**Interest Rate**	**Foreign Exchange rate and Gold**	**Credit (Investment-grade obligor)**	**Credit (Non-investment-grade obligor)**	**Equity**	**Precious Metals (except Gold)**	**Other Commodity**
< 1 year	0.00%	1.00%	5.00%	10.00%	6.00%	7.00%	10.00%
1 - 5 years	0.50%	5.00%	5.00%	10.00%	8.00%	7.00%	12.00%
> 5 years	1.50%	7.50%	5.00%	10.00%	10.00%	8.00%	15.00%

Source: The Federal Reserve Board, 2006

Add-on approaches are computationally fast and allow exposures to be pre-calculated and represented in simple grids, allowing for a quick look-up of the PFE of a new trade. Unfortunately, add-on approaches do not account for more subtle effects including:

- Transaction specifics;
- If the fair value of the transaction is far from zero;
- Netting;
- Collateral.

The Basel Accords allow netting to be incorporated, but under strict rules. For example, only 60% of the netting benefit can be used to offset the add-on (Gregory, 2012:158). There are more sophisticated add-on methodologies available, which the reader can find in publications by Rowe (1995) and Rowe and Mulholland (1999), although the increased complexity must be balanced against the Monte Carlo methodology, discussed in the next section.

2.7.2 Monte Carlo Methodology

Although the Monte Carlo methodology is the most complex and computationally intensive method to calculate exposure, it is completely generic and copes with the many complexities including transaction specifics, path dependency, netting and collateralisation, which the simple CEM struggles to capture. In the case of a high-dimensional netting set, it is the only method that can realistically incorporate a relatively large number of risk factors and their correlations (Gregory, 2012:159). In this section a general discussion of Monte Carlo simulation will be provided, supplemented by the methodology to simulate EE for an IRS.

2.7.2.1 General Monte Carlo Simulation Framework

Let X be a given random variable with $E(X) = \theta$, where the true value is unknown, and $Var(X) = \sigma^2$. In Monte Carlo simulation, N observations of X i.e. $\{X_i : i = 1, \dots, N\}$ is generated, given the distribution of X. The parameter θ is estimated by the sample mean of $\{X_1, \dots, X_N\}$, i.e. $\hat{\theta} = \frac{1}{N}\sum_{i=1}^{N} X_i$. This implies that $\hat{\theta}$ is an unbiased estimator for θ since

$$
\begin{aligned}
E(\hat{\theta}) &= E\left[\frac{1}{N}\sum_{i=1}^{N} X_i\right] \\
&= \frac{1}{N}\sum_{i=1}^{N} E(X_i)\,, \qquad i\ = 1, \dots, N \\
&= E(X) \\
&= \theta,
\end{aligned}
$$

with $Var(\hat{\theta}) = \frac{\sigma^2}{N}$. By the Law of Large Numbers given in Theorem 2.1, the estimate becomes more accurate as N, the number of simulations, increases (Van der Merwe, 2010:7; Glasserman, 2004:1-2; Rizzo, 2008:120).

Theorem 2.1 (The Law of Large Numbers) Let $X_1, \dots, X_N$ be independently identically distributed (i.i.d.) random variables with mean θ and variance σ^2. Then for any given $\delta > 0$, $P(|\hat{\theta} - \theta| > \delta\ ) \to 0\ as\ N \to \infty$ (Rice, 2007:175).

Therefore, the EE will increase in accuracy as the number of times the fair value of the IRS is estimated increases. The general Monte Carlo simulation method will be applied to the simulation of EE for an IRS in the following sections and will be expanded on in Chapter 3.

2.7.2.2 Valuation of an Interest rate Swap

To simulate the EE for a plain vanilla IRS, the IRS needs to be valued at certain points of time in the future. This section will discuss the methodology for plain vanilla IRS valuation as presented by Hull (2009:159-163), Lesniewsky (2008:6-8) and Ahlberg (2013:42).

Let $T_1 < \dots < T_{n_{fixed}}$ denote the coupon dates of the swap and let $T_0 = 0$. The present value (PV) of the interest payments on the fixed leg of a swap is calculated as the sum of all future cash flows

$$PV_{fixed} = \sum_{i=1}^{n_{fixed}} \alpha_i C_{fixed} P(0, T_i), \tag{2.20}$$

where C_{fixed} is the fixed coupon rate, $P(0, T_i)$ is price at time zero of a zero coupon bond yielding one unit of money at time T_i representing the discount factor for the ith coupon date and α_i is the day-count fraction applying to each period. This formula can be rewritten as

$$PV_{fixed} = C_{fixed} L, \tag{2.21}$$

where

$$L = \sum_{i=1}^{n_{fixed}} \alpha_i P(0, T_i), \tag{2.22}$$

is called the $DV01$ of the swap. For the floating leg, which will be tied to a rate such as JIBAR, it can be formulated as

$$PV_{float} = \sum_{i=1}^{n_{float}} \delta_i L_i P(0, T_i), \tag{2.23}$$

where

$$\begin{aligned} L_i &= F(T_{i-1}, T_i) \\ &= \frac{R(0, T_i)T_i - R(0, T_{i-1})T_{i-1}}{T_i - T_{i-1}} \\ &= \frac{1}{\delta_i}\left(\frac{1}{P(T_{i-1}, T_i)} - 1\right), \end{aligned} \tag{2.24}$$

is the forward rate for settlement at T_{i-1}, $P(0, T_i)$ is the discount factor for the ith coupon date and δ_i is the day-count fraction applying to each period between T_{i-1} and T_i. It is important to note that it is possible to write

$$PV_{float} = 1 - P(0, T_{Mat}), \tag{2.25}$$

where T_{Mat} denotes the maturity of the swap and expresses the fact that a spot settled floating rate bond, paying JIBAR and repaying the principal at maturity, is always valued at par. To prove Equation 2.22 is simple:

$$\begin{aligned} PV_{float} &= \sum_{i=1}^{n_{float}} \delta_i L_i P(0, T_i) \\ &= \sum_{i=1}^{n_{float}} \left(\frac{1}{P(T_{i-1}, T_i)} - 1\right) P(0, T_i) \\ &= \sum_{i=1}^{n_{float}} [P(0, T_{i-1}) - P(0, T_i)] \\ &= 1 - P(0, T_{n_{float}}) \end{aligned}$$

The PV of a payer swap is the difference between PV_{fixed} and PV_{float} (in this order):

$$PV_{swap} = PV_{fixed} - PV_{float}, \tag{2.26}$$

implying the PV of a receiver swap is a matter of changing the sign. At inception the IRS will be priced to ensure the PV is zero. In the derivation it was assumed that no spread would be added to the floating leg of the IRS. If a spread is to be added, a simple modification can be made, which is discussed in section 3.5.2.1.

2.7.2.3 The Interest Rate Model

To estimate the value of the IRS to calculate the EE, the term structure of the interest rate will need to be estimated. According to Svoboda (2002:208), the choice of model in the South African market is largely driven by availability and reliability of market data. The South African interest rate market is thin, which makes implementing sophisticated models such as the Libor Market Model and the Heath, Jarrow and Morton Model difficult and the results questionable.

Therefore, the stochastic interest rate model chosen to model the term structure is the Vasicek (One Factor) model proposed by Vasicek (1977), which will only be referred to as the Vasicek model hereafter. Hull and White (1990) explored extensions of the model that provides an exact fit to the initial term structure, which is not necessary for the purposes of this paper and therefore the Vasicek model will be an adequate choice. The Vasicek model also has the characteristic of mean reversion and it is possible to find analytical formulas for bond and option prices. Although, difficulties arise in low interest rate countries, since a draw back of the Vasicek model is negative rates, it will not be a factor in the South African context. Before the Vasicek model is to be described in detail, background on one-factor short rate models will be given.

The short rate at time t, $r(t)$, is the rate that applies to the infinitesimally short period of time at time t and is also referred to as the instantaneous short rate. In a traditional risk-neutral world considered here, in a very short period of time between t and $t + \delta t$, investors earn on average $r(t)\delta t$ (Hull, 2009:673).

From the equivalent martingale measure result defined in Appendix A, it follows that in a risk-neutral world the value at time t of an interest rate derivative that provides a payoff of f_T at time T is

$$f_t = E^{\mathbb{Q}}\left(e^{-\bar{r}(T-t)} \cdot f_T\right), \tag{2.27}$$

where $\bar{r}$ is the average value of r over the interval between t and T, $E^{\mathbb{Q}}$ denotes the expected value in the traditional risk-neutral world. Hence,

$$P(t,T) = E^{\mathbb{Q}}\left(e^{-\bar{r}(T-t)} \cdot 1\right), \tag{2.28}$$

where $P(t,T)$ is the price, at time t of a zero coupon bond of unit face value maturing at time T, where $t \leq T$ and $P(T,T) = 1$.

If $R(t,T)$ is the continuously compounded interest rate at time t for a period of $T-t$, which is defined in Chapter 2.2, then

$$P(t,T) = 1 \cdot e^{-R(t,T)(T-t)}, \tag{2.29}$$

so that

$$\begin{aligned} R(t,T) &= -\frac{1}{T-t}\ln P(t,T) \\ &= -\frac{1}{T-t}\ln E^{\mathbb{Q}}\left(e^{-\bar{r}(T-t)} \cdot 1\right), \end{aligned} \tag{2.30}$$

from Equation 2.28.

(Park, 2004:3; Hull, 2009:674)

This equation enables the term structure of interest rates at any given time to be obtained from the value of r at that time and the risk-neutral process of r. Once the process for r is defined, the initial zero curve and its evolution through time is completely known (Hull, 2009:674). With this information as background it is now possible to define the Vasicek model.

2.7.2.4 The Vasicek One-Factor Model

As previously mentioned, the Vasicek model was first introduced by Vasicek in 1977 and was the first continuous time interest rate model to gain wide spread acceptance. In the specific one-factor Vasicek model, the underlying stochastic process for the short rate $r(t)$ is given by

$$dr(t) = a[b - r(t)]dt + \sigma dW(t), \tag{2.31}$$

where

- $W(t)$ represents a standard Brownian motion;
- $a > 0$, is a constant representing the rate of mean reversion;
- σ is a constant representing the standard deviation;
- b is a constant rate to which the short rate is pulled at rate a.

According to Svoboda (2002:2), Vasicek makes three key assumptions. The first assumption is that the current spot interest rate is known with certainty. However, the subsequent values of the spot rate are not known. Also assume that the short rate $r(t)$ follows a Markov process. That is, given the current value, future developments of the spot rate are independent of past movements. The second assumption is that the value of the zero coupon bond, $P(t,T)$, is fully determined by the time t assessment of $\{r(t^*): t < t^* < T\}$, the segment of the spot rate over the remaining term of the bond. Moreover, the development of the spot rate is fully determined by the current value of $r(t)$. The third assumption is that the market is efficient. This implies that there are no transaction costs, investors receive all information simultaneously, all investors are rational and that a riskless arbitrage profit is not possible.

Svoboda (2002: 5-6) goes further and shows that from Equation 2.29, zero coupon bond prices at time t of unit face value in the Vasicek model are given by

$$\begin{aligned} P(t,T) &= e^{-R(t,T)(T-t)} \\ &= A(t,T)e^{-B(t,T)r(t)}, \end{aligned} \tag{2.32}$$

with

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a} \tag{2.33}$$

and

$$A(t,T) = \exp\left[\frac{(B(t,T) - T + t)(a^2 b - \frac{\sigma^2}{2})}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right] \tag{2.34}$$

Alternatively, the spot rate can be calculated from equation 2.30 as follows,

$$R(t,T) = -\frac{1}{T-t}\ln A(t,T) + \frac{1}{T-t}B(t,T)r(t) \tag{2.35}$$

(Hull, 2009:676; Svoboda, 2002:6; Vasicek, 1977:181).

2.7.2.5 A Note On Model Calibration

To use the Vasicek model in the pricing of derivatives it needs to be calibrated to represent the characteristics of the term structure in the market. There are three parameters in the Vasicek model that determine the evolution of the short rate.

The model parameter b is chosen as the long-term rate to which the short rate is pulled at a rate of a. The mean reversion rate a and volatility σ are calibrated to the prices of interest rate derivatives such as options, caps and floors or swaptions by means of optimisation. A popular goodness-of-fit measure is

$$\sum_{i=1}^{n} (U_i - V_i)^2, \tag{2.36}$$

where U_i is the market price of the ith calibrating instrument and V_i is the price given by the model for this instrument (Hull, 2009:696-697). The parameters are then chosen to minimise the goodness-of-fit measure.

In the absence of liquid calibrating instruments, the usual route is to rely on historical interest rate data, either explicitly or implicitly. These usually take the form of spot rate time series of different maturities (Ahlberg, 2013:45). There are numerous other methods to calibrate the Vasicek model, but the focus of this paper is not on model calibration and the reader is referenced to Park (2004) and Svoboda (2002) for further reading.

2.8 MITIGATING COUNTERPARTY CREDIT RISK

There are various means to reduce CCR. Netting and collateral have been common methods to achieve this. These methods are of a bilateral nature and therefore aim to reduce the risk for both parties. In the event of a default, netting allows the offset of exposure amounts owed to and by the counterparty. Netting is finite and greatly dependent on the type of underlying transactions involved.

In theory, collateral can reduce the CCR even further to the point of elimination but it creates significant operational costs and other risks, such as liquidity risk and legal risk. Netting will be discussed in greater detail in the next section, followed by a discussion on collateral summarised from Gregory (2012).

2.8.1 Netting

When a counterparty defaults, the market needs a mechanism whereby participants can replace their positions with other counterparties. Furthermore, it is desirable for an institution to be able to offset what it owes to the defaulted counterparty against what they themselves are owed from the counterparty. There are two mechanisms to facilitate this, payment netting and closeout netting. Payment netting gives the institution the ability to net cash flows occurring on the same day, whilst closeout netting allows for the termination of all transaction values, both in favour and against (Gregory, 2012:46-47). This typically relates to CCR.

2.8.1.1 Netting sets

The concept of a netting set corresponds to a set of trades that can be legally netted together in the event of a default. A netting set can be a single trade and there may be more than one netting set per counterparty. Exposure will always be additive across netting sets. It is important to point out that within the netting set, quantities such as EE and CVA are non-additive (Gregory, 2012:49-50).

2.8.1.2 The Impact of Netting on Exposure

Since netting allows the future values of trades to offset one another, then the aggregate effect of all trades needs to be considered (Gregory, 2012:139). The impact of netting on future exposure can be illustrated in Figure 2.5 with two transactions, PV 1 and PV 2. PV 1 goes from positive to negative, whilst PV 2 is only positive. With no netting agreement, exposures are additive, since the positions do not offset one another. If netting is allowed, the values can be added at the netting set level before calculating the exposure and therefore shows less exposure at the points in the future.

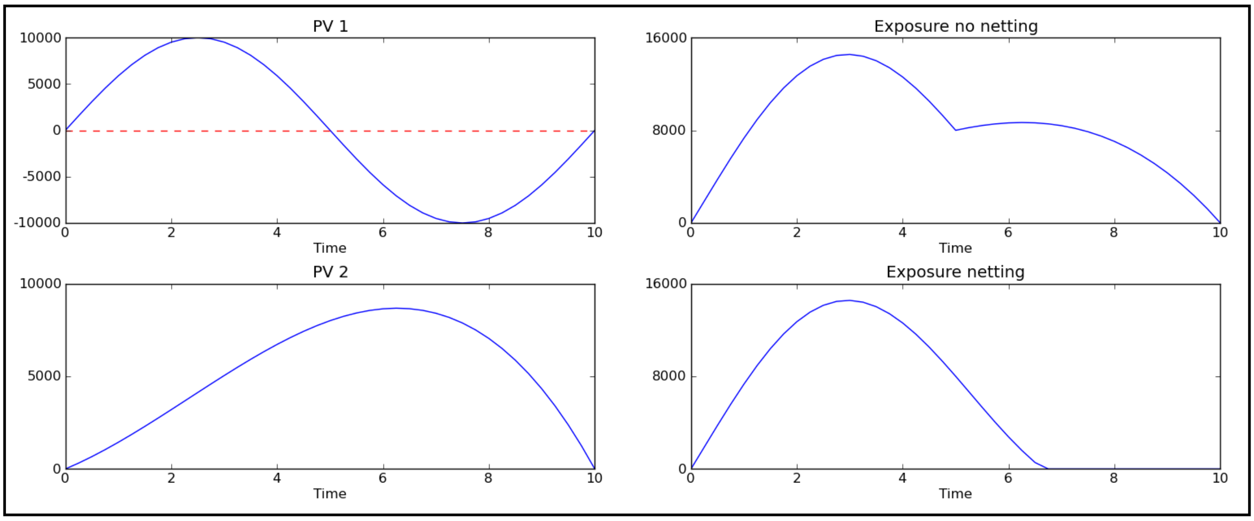

Figure 2.5: The Impact of Netting on Exposure

Source: Ahlberg, 2013:13

2.8.2 Collateral

Collateralisation can reduce credit exposure even further. Collateral agreements may often be negotiated prior to any trading activity between counterparties or may be agreed or updated prior to any increase in trading volume or other agreements (Gregory, 2012:59).

2.8.2.1 The Basics of Collateralisation

The idea of collateralisation is simple and is illustrated in Figure 2.6. Consider a transaction between two parties A and B. Party A makes a MtM profit whilst party B makes a corresponding MtM loss. Party B then posts some form of collateral to party A to mitigate the credit exposure that arises due to the positive MtM. The collateral may be cash or other securities that will be specified before the initiation of the contract. Since collateral agreements are often bilateral, collateral must be returned or posted in the opposite direction when exposure decreases (Gregory, 2012:61).

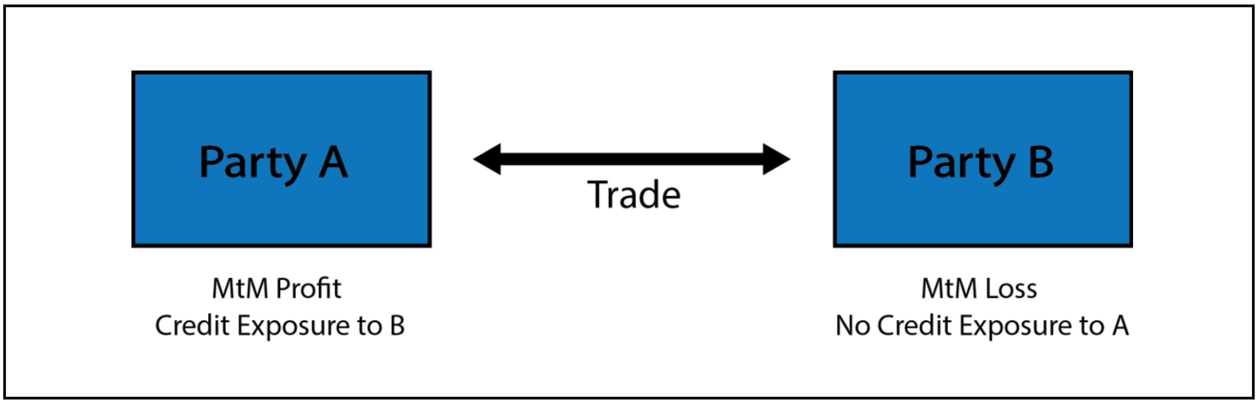

Figure 2.6: Illustration of the basic principle of collateralisation

Source: Gregory, 2012:61

2.8.2.2 The Impact of Collateral on Exposure

The impact of collateral on a typical exposure profile is shown in Figure 2.7. There are two reasons why collateral cannot mitigate exposure perfectly. Firstly, the presence of a threshold ensures that a certain amount of exposure cannot be collateralised. Secondly, the delay in receiving collateral and constraints such as a minimum transfer amount create a discrete effect illustrated by the grey bars in Figure 2.7 (Gregory, 2012:64).

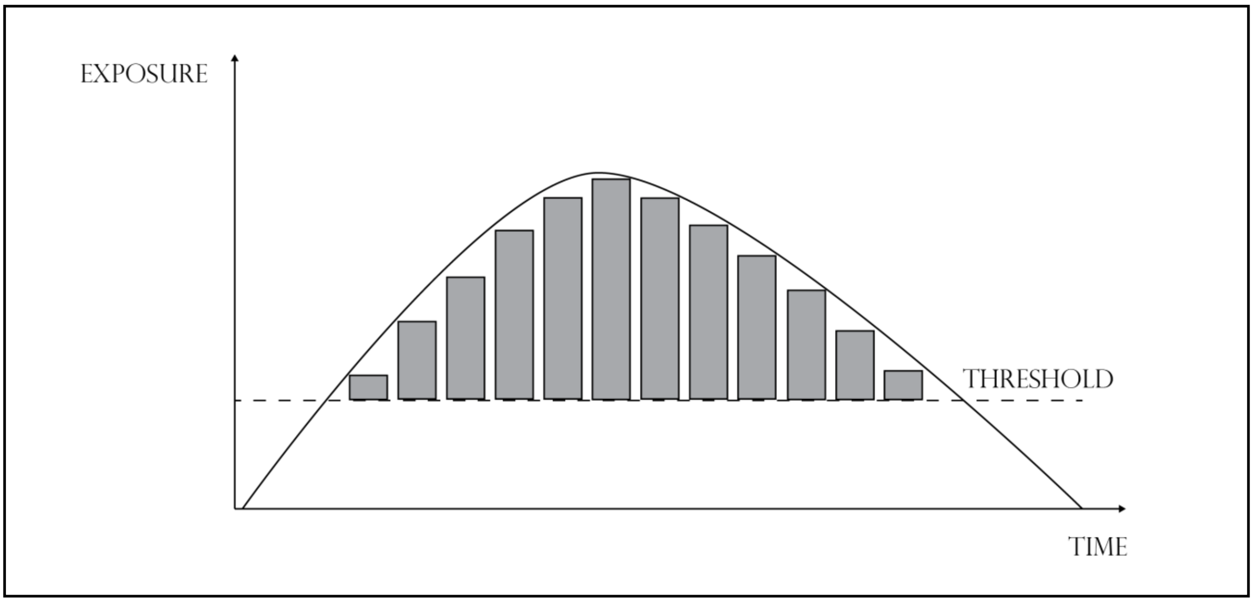

Figure 2.7: The impact of collateralisation on exposure

Source: Gregory, 2012:64

2.9 SUMMARY

A formula to estimate CVA was derived in the first section, followed by detailed discussions on each component of CVA such as PD, LGD and EE. Thereafter, two methods to mitigate CCR, netting and collateralisation were discussed.

In the following chapters the literature discussed in this chapter will be applied to the calculation of the EE of the IRS. In Chapter 3 the research methodology will be described, whilst Chapter 4 will represent the results of the study. Chapter 5 will offer a conclusion and open questions for further research.

CHAPTER 3
RESEARCH METHODOLOGY

3.1 INTRODUCTION

This chapter will represent the methodology used to calculate credit valuation adjustment (CVA) for a plain vanilla interest rate swap (IRS). The focus will be on the calculation of the expected exposure (EE) element, whilst other less important (to this study) elements such as the probability of default (PD) and loss given default (LGD) will be discussed in lesser detail. These methodologies will then be used to find the results represented in Chapter 4.

3.2 CREDIT VALUATION ADJUSTMENT

Although the quantification of EE is the focus of this paper, the CVA will also be presented as a result in Chapter 4. CVA was discussed in section 2.3. The formula used to calculate CVA is

$$\begin{aligned} CVA(t,T) &= (1-\Theta)\left[\int_t^T EE(u,T)B(t,u)dF(t,u)\right] \\ &\approx (LGD)\sum_{i=1}^{N} DF_{t_i}EE_{t_i}PD(t_{i-1},t_i), \end{aligned} \tag{3.1}$$

where $(1-\Theta) = LGD$, DF is the relevant risk-free discount factor, EE is the non-discounted expected exposure on the relevant dates and PD is the probability of default between relevant dates. According to Gregory (2012:245), the accuracy of the estimation of EE can be improved by transforming

$$EE_{t_i} \approx \frac{(EE_{t_{i-1}} + EE_{t_i})}{2}$$

and

$$DF_{t_i} \approx \frac{(DF_{t_{i-1}} + DF_{t_i})}{2}$$

This variance reduction technique is referred to as Stratified Sampling and more on variance reduction techniques can be found in Glasserman (2004) and Rizzo (2008). The discount factor for the calculation of CVA in this study will be derived from a constant annual term structure of 5.50%. This is a simplifying assumption to assist with the comparison between CVA calculation methodologies and the reader should note that in practise the current term structure should be used.

3.3 PROBABILITY OF DEFAULT

The PD will be assumed as constant over time in this study. The Financial Services Board of South Africa (FSB) concluded in the third South African Quantitative Impact Study (SA QIS3) that if the PD in the following year, PD_1, is known and is assumed to be constant over time that the probability of default in year t can be estimated by

$$PD_t = PD_1(1 - PD_1)^{t-1} \tag{3.2}$$

In this paper a probability of default of 1% per quarter will be assumed.

3.4 RECOVERY RATES AND LOSS GIVEN DEFAULT

Since the focus of this paper is not on estimating the loss given default fraction, it is assumed that the LGD will be constant over the time periods. The Financial Services Board of South Africa (FSB) concluded in the third South African Quantitative Impact Study (SA QIS3) that an average LGD of 60% is sufficient. Therefore, the constant LGD used in this study will be 60%.

3.5 QUANTIFYING CREDIT EXPOSURE

In the following sections the methodology for quantifying exposure with the current exposure method (CEM) and the Monte Carlo Method, discussed in section 2.7 will be described. All calculations are done assuming a notional (Not) of one. The exposure calculations will then be used to calculate CVA as described in sections 2.3 and 3.2.

3.5.1 Mark-to-Market Plus Add-on (Current Exposure Method)

To calculate the exposure with the CEM, the current value,V_0, of the IRS will be calculated with the swap curve and used in the following formula corresponding to Equation 2.19

$$EAD = \max(V_0, 0) + Not \cdot CCF \tag{3.3}$$

This formula gives the exposure at default (EAD). The credit conversion factor (CCF) will be assumed as given by Basel II in Table 3.1:

Table 3.1: Conversion Factors for Interest Rate Derivatives

	Type
Remaining Maturity (Years)	**Interest Rate**
< 1 year	0.00%
1 - 5 years	0.50%
> 5 years	1.50%

Source: The Federal Reserve Board, 2006

There are two methods by which the CEM can be expanded when calculating the CVA. Therefore, call the method described above Method 1 and the expanded methods, Method 2 and Method 3. With Method 2, the EAD is calculated in the same manner as in Method 1, except that the number of time steps in the calculation is now only one. The formula for CVA is then

$$CVA = EAD \cdot (1 - (1 - PD)^{D}) \cdot LGD, \tag{3.4}$$

where D is the estimated duration. The estimated duration is calculated as 50% of the time to maturity.

In Method 3 EAD at time $t = 0$ is also calculated in the same manner as the normal CEM. The EAD is then used to estimate an EE profile by decreasing the EAD linearly to zero for the remainder of the IRS tenor. The CVA is then calculated with the same formula derived in Chapter 2.

These expansions to the CEM were created in an effort to increase the accuracy of the CEM approach. CVA will be calculated with all three methods and then compared to the Monte Carlo method.

3.5.2 Monte Carlo Methodology

In the following subsections the methodology for simulating EE with the means of Monte Carlo simulation will be discussed.

3.5.2.1 Valuation of an Interest Rate Swap

The theory of the valuation of an IRS was discussed in section 2.7.2.2. Once the parameters of the IRS, namely the swap rate, maturity, payment frequency, the notional amount and the floating rates are known, the price of the IRS at $t = 0$ can be calculated. The exposure of the IRS at the future coupon dates can then be estimated. The floating rates are derived from the interest rate curve implemented as explained in the next section. The following equation will be used to value the fixed leg of the IRS

$$PV_{fixed} = \sum_{i=1}^{n_{fixed}} \alpha_i C_{fixed} P(0, T_i) \tag{3.5}$$

This corresponds to equation 2.20. To value the floating leg of the IRS the following equation will be used

$$PV_{float} = \sum_{i=1}^{n_{float}} \delta_i L_i P(0, T_i), \tag{3.6}$$

where

$$L_i = F(T_{i-1}, T_i),$$

is the forward rate for settlement at T_{i-1}. According to Hull (2009:83) the forward rates can be calculated from the zero rates as

$$\begin{aligned} F(T_{i-1}, T_i) &= \frac{R(0,T_i)T_i - R(0,T_{i-1})T_{i-1}}{T_i - T_{i-1}} \\ &= R(0,T_i) + [R(0,T_i) - R(0,T_{i-1})]\frac{T_{i-1}}{T_i - T_{i-1}}, \end{aligned} \tag{3.7}$$

which is more relevant than using the discount factor since it allows for the inclusion of a spread to the floating rate. In this study it will be assumed that the day count fractions α_i and δ_i are equal and no provision is made for a specific day count convention.

3.5.2.2 The Interest Rate Model

To model the EE if an IRS, the interest rate will need to be simulated to calculate the floating rate coupons, described in the previous section. The algorithm to simulate the Vasicek One-Factor model will be discussed below.

Let $r(t)$ be the short rate described in section 2.7.2.3. Then the Vasicek model describes $r(t)$ by the following equation corresponding to equation 2.31

$$dr = a[b - r]dt + \sigma dW \tag{3.8}$$

Equation 3.8 can be discretised and written as

$$\delta r = r(t + \delta t) - r(t) = a\big(b - r(t)\big)\delta t + \sigma Z\sqrt{\delta t}, \tag{3.9}$$

so that

$$r(t + \delta t) = r(t) + a\big(b - r(t)\big)\delta t + \ \sigma Z\sqrt{\delta t}, \tag{3.10}$$

where $Z \sim \mathrm{N}(0,1)$, a, b and σ are the model parameters described in section 2.7.2.4.

If the model parameters a, b and σ are known, as well as the current spot rate $R(0,T)$, the following algorithm can be used to generate $r(t)$ for $= 0, \delta t, 2\delta t, 3\delta t, \ldots$.

Algorithm 3.1 (Steps to simulate the Vasicek short rate)

1. *From equation 3.10,*

$$r(t + \delta t) = r(t) + a(b - r(t)))\delta t + \ \sigma Z\sqrt{\delta t} \tag{3.11}$$

2. *Let* $r(0) = R(0)$.

3. *Generate a standard normal random variable Z_1. By equation 3.11*

$$r(\delta t) = r(0 + \delta t) = r(0) + a(b - r(0))\delta t + \sigma Z_1 \sqrt{\delta t}$$

4. *Repeat steps 1 to 3 to generate $r(t)$ for $t = 0, \delta t, 2\delta t, 3\delta t, \dots$*

Now that the short rates are generated, the zero curve can be calculated from equation 2.35. In this paper the model parameters a, b and σ will not be calibrated, but rather chosen to approximately fit the shape of the swap curve prevailing in the South African market. The choices were $a = 0.6, b = 0.07$ and $\sigma = 0.11$ with $r(0)$ = 5.1% corresponding to the three month JIBAR rate on 26 September 2014. The term structure is represented in Figure 3.1 below. This algorithm was implemented in the **R** statistical computer software program (R Development Core Team, 2014).

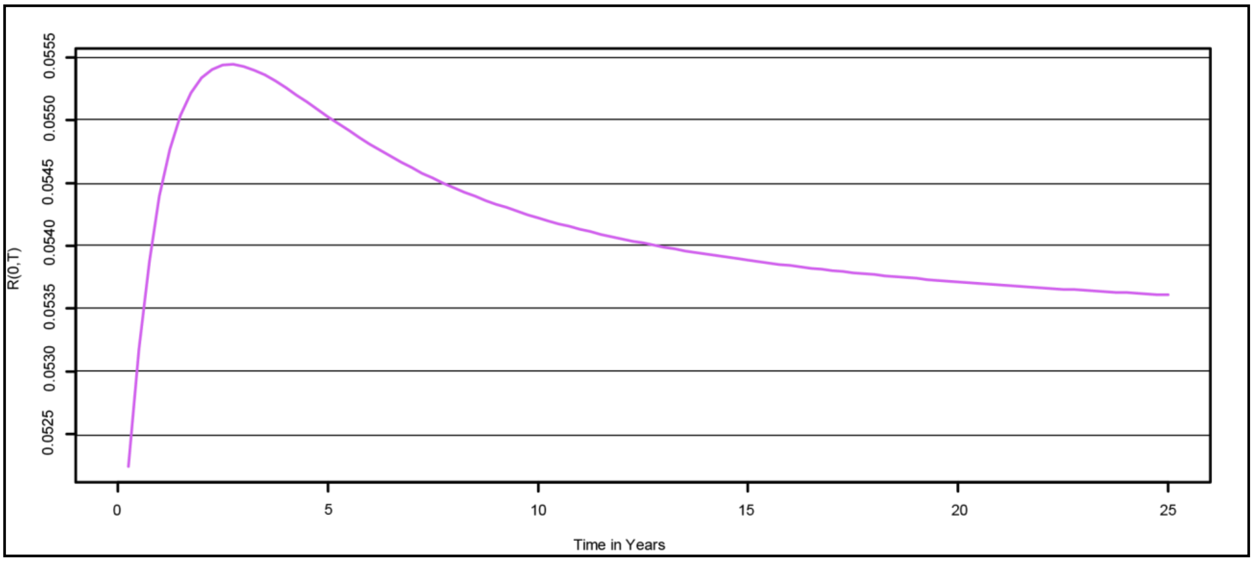

Figure 3.1: The initial Vasicek term structure, $R(0,T)$, for $T = 0, 0.25, 0.5, \dots, 25$ years. The parameters are $a = 0.6, b = 0.07$ and $\sigma = 0.11$ with $r(0)$ = 5.1%.

3.5.2.3 Exposure

The methodology concerning the valuation and estimation of IRS exposure was discussed and now the following equation

$$EE_{t_i} = \frac{1}{N} \sum_{i=1}^{N} \max(V_{t_i}, 0), \tag{3.12}$$

corresponding to equation 2.18 will be used to estimate EE. As an addition, potential future exposure (PFE) will also be illustrated parallel to EE.

3.7 COMPARISON METHODOLOGY

Since it will be very difficult to estimate the accuracy of the CEM methods with a statistical quantity, it will be assumed that the Monte Carlo method is 100% accurate. The exposure profiles of the CEM methods will then be compared to the Monte Carlo method EE value in absolute terms. Thereafter, the CVA will be calculated with each method and then compared to the Monte Carlo method CVA.

3.8 SUMMARY

This chapter discussed the methodology to implement the theory discussed in Chapter 2. The method to calculate each component of CVA such as PD, LGD and EE was discussed followed by the method used to compare the CEM with the Monte Carlo method. In the following chapter the results will be represented and discussed.

CHAPTER 4
RESULTS

4.1 INTRODUCTION

In this chapter the results will be represented and discussed. The results were obtained by using the methodology described in Chapter 3. In the following sections the details of the various interest rate swaps (IRS) used to obtain the results will be given, followed by the expected exposure calculations with the Current Exposure Method (CEM) methods and the Monte Carlo methodology. Finally, the exposure profiles will be used to calculate the credit valuation adjustment (CVA) to compare the adjusted fair values of the two methods.

4.2 INTEREST RATE SWAP DETAILS

There were nine different IRS's used to obtain the results for this study. The details of these IRS's are given in Table 4.1. For simplicity, it is assumed that all the IRS's are floating payer swaps and that the payment frequency is quarterly throughout.

Table 4.1: Details of the IRS's used in the study

Swap	Swap Rate	Tenor (Years)	Spread	Current Fair Value
1	0.04	2	0	-0.02951
2		10	0	-0.11259
3		25	0	-0.19224
4	0.08	2	0	0.04691
5		10	0	0.19338
6		25	0	0.35329
7	0.12	2	0	0.12223
8		10	0	0.50156
9		25	0	0.89882

It was assumed that the notional principal of each swap is one. To calculate the current fair value of the swaps, the initial term structure given in section 3.5.2.2 was used. These values are given in the last column of Table 4.1. It is evident that the various swap rates and the tenor of each swap greatly effects the current value of the IRS. If the swap rate increases from 4.00% to 8.00% for 10-Year swaps, the current value increases from -11.26% of the notional to 19.34% of the notional amount, respectively. If the swap rate is increased futher to 12.00% the current value increases to 50.16% of the notional amount. Further, if the tenor of the IRS with a 8.00% swap rate is increased the current value becomes 4.69% for a 2-Year tenor, 19.34% for a 10-Year swap and 35.33% for a 25-Year swap.

4.3 EXPECTED EXPOSURE

The theory regarding the quantification of exposure was discussed in Chapter 2.7. Before the exposure profiles for the various methods will be compared, the expected exposure (EE) and potential future exposure (PFE) estimated by the Monte Carlo method for each IRS is represented and discussed in Figure 4.2, Figure 4.3 and Figure 4.4 below. The tables representing the values of the EE are given in Appendix B.

The Vasicek model was used to simulate 10 000 interest rate paths for each swap to estimate the EE. The model parameters that was used are $a = 0.6, b = 0.07$ and $\sigma = 0.11$ with $r(0)$ = 5.1% corresponding to the three month JIBAR rate on 26 September 2014. This was accomplished by simulating the short rate for each day of the year, assuming 252 trading days in a year and then calculating the term structure to maturity on each payment date in the future. The value of the swap was then calculated on each payment date using the simulated term structures. These values are illustrated in Figure 4.1 using swap number 2 in Table 4.1.

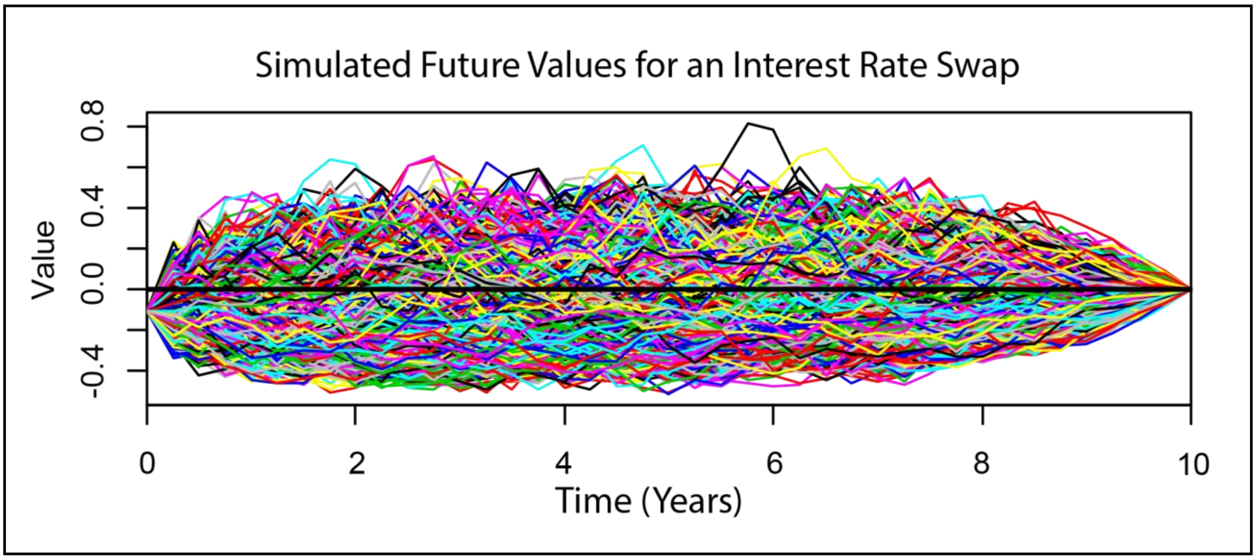

Figure 4.1: Simulated future values for an Interest rate swap with Swap Rate = 4% and Tenor = 10 years

Each of the lines in Figure 4.1 represents one simulation of the value of the swap on each quarterly payment date in the future from $t = 0$ to $t = 10$. Such a simulation was done for each swap in the Table 4.1 and these values were then used to calculate the EE and PFE.

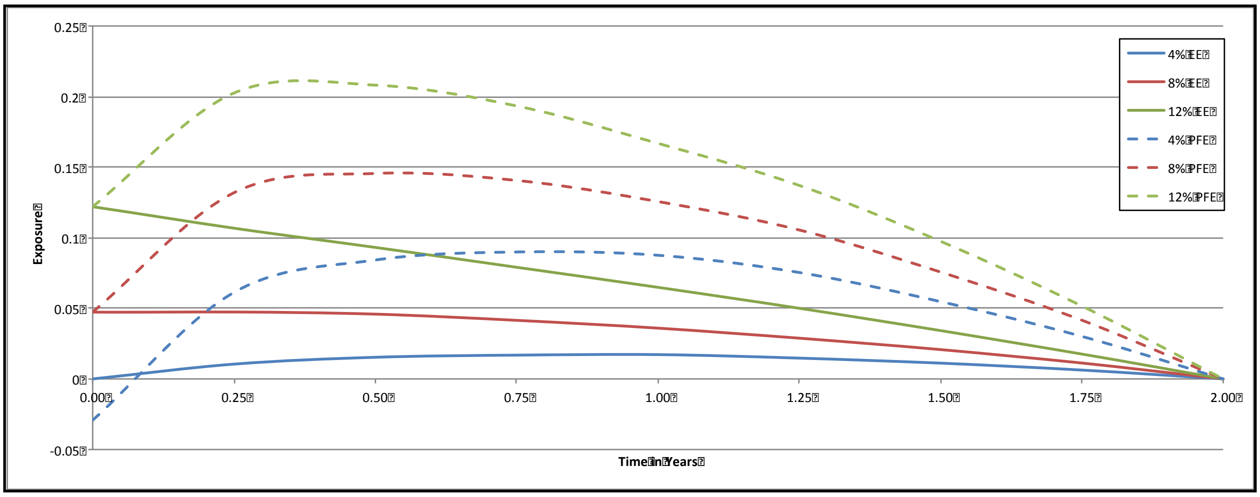

Figure 4.2: Graphical representation of Expected Exposure and Potential Future Exposure for 2-year swaps

Figure 4.2 illustrates the EE and PFE for 2-Year IRS's. The usual shape of the EE curve is only evident in the 4.00% swap, where the EE increases to a maximum at approximately a third of the tenor and thereafter decreases to zero, since the current values of the 8.00% and 12.00% swaps were too far from zero. Therefore, it seems that the EE for the 8.00% and 12.00% is decreasing from $t = 0$ to $t = 2$. The 95% PFE for each swap in Figure 4.2 has the desired shape, although the starting values for the 8.00% and 12.00% swaps are high above zero. For the 8.00% IRS, the PFE can be interpreted that, with 95% certainty, the entity will not lose more than 14.55% of the notional amount if the counterparty defaults between now and maturity. Note that the EE and PFE of the 4.00% IRS does not originate from the same point, since the current value of the IRS is negative. By definition, EE cannot be negative, whilst PFE can be.

These characteristics are also evident in Figure 4.3 and in Figure 4.4, where the tenor for each IRS is 10 years and 25 years respectively. It can be seen that the EE and the PFE increases as the swap rate and the tenor of each swap increases.

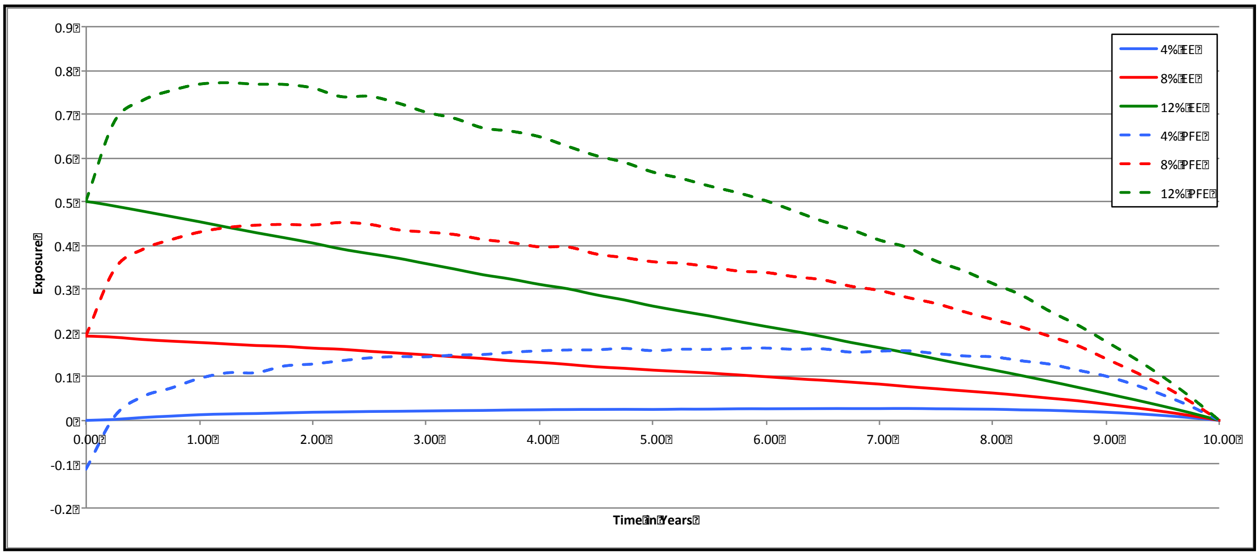

Figure 4.3: Graphical representation of Expected Exposure and Potential Future Exposure for 10-year swaps

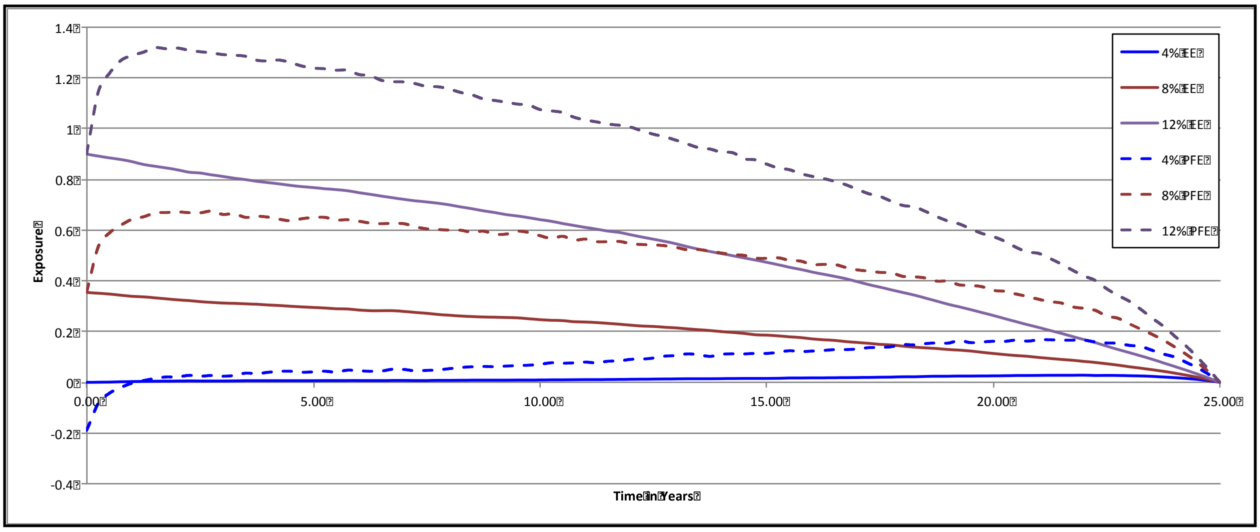

Figure 4.4: Graphical representation of Expected Exposure and Potential Future Exposure for 25-year swaps

It is now possible to compare the Monte Carlo method to the three methods under CEM. The values are tabulated in Appendix B. Figure 4.5, Figure 4.6 and Figure 4.7 represents the exposure profiles for the Monte Carlo method compared to the CEM profiles for the 10-Year IRS for each swap rate. The figures for the 2-Year and 25-Year IRS are represented in Appendix B. The exposure for CEM Method 2 will not be presented in the figures since it is only one time step.

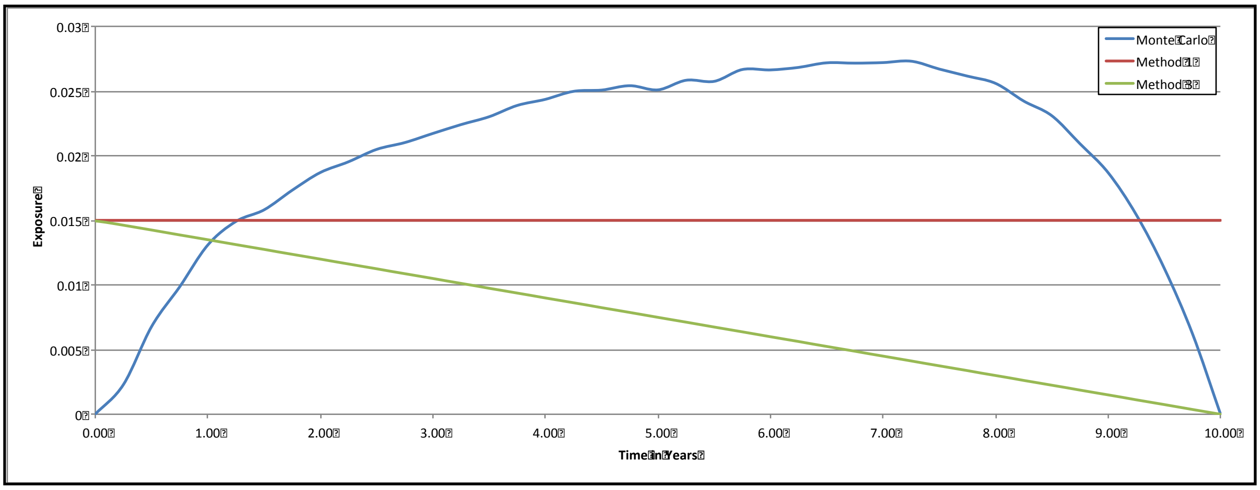

Figure 4.5: Exposure Profiles for the 4% 10-Year IRS for different methods

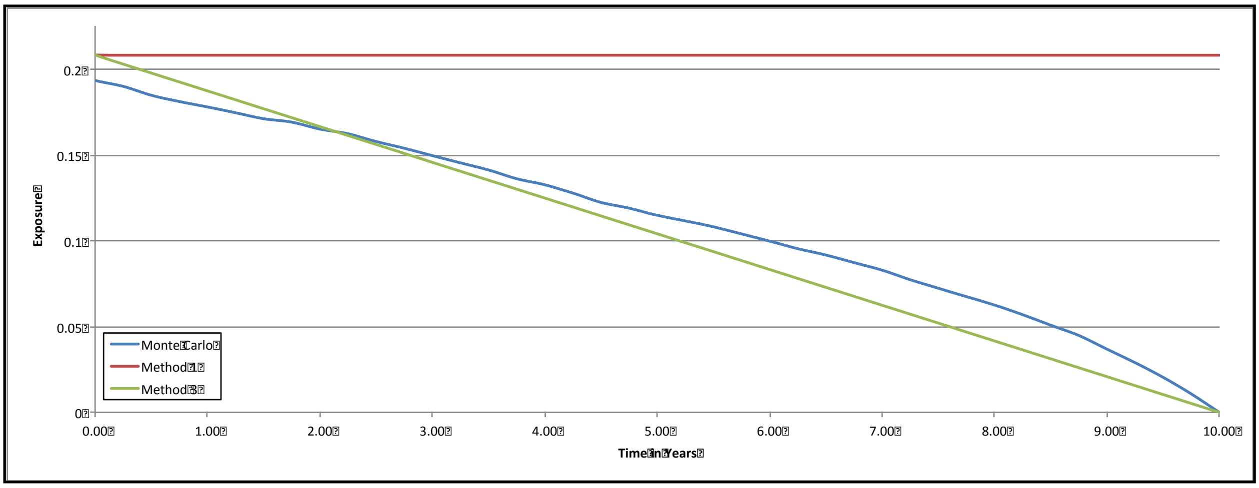

Figure 4.6: Exposure Profiles for the 8% 10-Year IRS for different methods

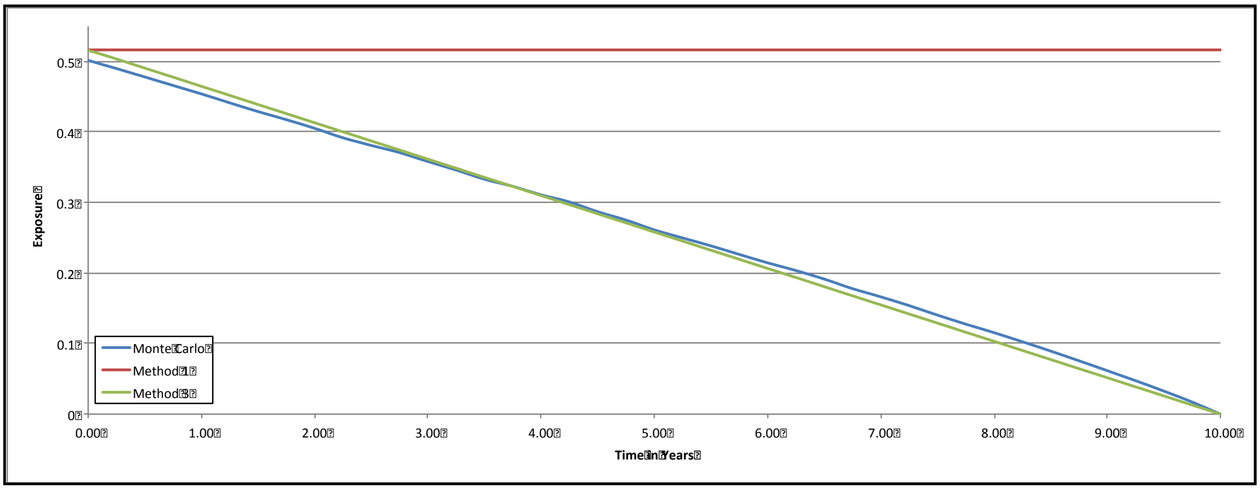

Figure 4.7: Exposure Profiles for the 12% 10-Year IRS for different methods

It can be seen in Figure 4.5 that the there are relatively large differences between the Monte Carlo method, Method 1 and Method 3 for the 4.00% IRS. The 8.00% IRS exposure profiles for the Monte Carlo method and Method 3 is much closer, shown in Figure 4.6, whilst the exposure profile for Method 1 is clearly deviating from the Monte Carlo method. This trend can be seen further in Figure 4.7 with the 12% IRS. Method 3 and the Monte Carlo Method are almost identical, whilst Method 1 deviates from the Monte Carlo method.

Therefore, the higher the current value of the IRS is, the better Method 3 compares to the Monte Carlo method. Similar trends are evident for the 2-Year IRS and the 25-Year IRS. The figures are represented in Appendix B. One can now ask whether the Monte Carlo method for estimating EE for smaller non-financial organisations is necessary, considering the similarities with Method 3. The question can be answered by calculating the CVA.

4.4 CREDIT VALUATION ADJUSTMENT

The exposure profiles represented in the previous sections will now be used to calculate the CVA. Table 4.2 represents the CVA calculated with the exposure estimated with the CEM Method 1, Method 2 and Method 3, as well as the CVA calculated with the Monte Carlo Method.

Table 4.2: Credit Valuation Adjustments

Credit Valuation Adjustment						
Swap Rate	**Tenor in Years**	**Current Fair Value**	**Monte Carlo**	**CEM: Method 1**	**CEM: Method 2**	**CEM: Method 3**
0.04	2	-0.02951	0.00052	0.00025	0.00012	0.00013
	10	-0.11259	0.00359	0.00278	0.00164	0.00153
	25	-0.19224	0.00305	0.00450	0.00355	0.00285
0.08	2	0.04691	0.00146	0.00264	0.00123	0.00135
	10	0.19338	0.02145	0.03779	0.02277	0.02099
	25	0.35329	0.07279	0.11061	0.08728	0.06987
0.12	2	0.12223	0.00291	0.00646	0.00301	0.00331
	10	0.50156	0.05048	0.09368	0.05644	0.05203
	25	0.89882	0.18650	0.27444	0.21657	0.17336

It can be seen in Table 4.2 that the value of CVA for all methods are increasing, as the tenor of the swaps increase, for each swap rate. If it can be assumed, as it will be in this study, that the Monte Carlo method is 100% accurate, it is evident that Method 1 relatively over estimates CVA. A 10-Year IRS with a swap rate of 8.00% has a CVA of 3.78% of the notional value when Method 1 is used. The same IRS has a CVA of 2.15% when the Monte Carlo method is used. This pattern is evident for all the various swap rates and tenors. Method 2 fairs relatively better compared to Method 1.

Method 3 is clearly the best CEM method. A 10-Year IRS with a swap rate of 8.00% has a CVA of 2.01% of the notional value, whilst the same IRS has a CVA of 2.15% when the Monte Carlo method is used. This pattern is not so clear when the current value of the swap is close to zero or negative. This can be seen when comparing CVA for the 4% IRS. For the 10-Year 4% IRS the CVA calculated with Method 3 is 0.013%, whilst the CVA calculated with the Monte Carlo method for the same IRS is 0.052%. The reader should keep in mind that although the difference seems small, the notional amount could be very large, causing a small difference in percentage to be a large difference in monetary terms.

4.5 CONCLUSION

Therefore, making a simple modification to the CEM such as Method 3, it is possible to improve the accuracy of the add-on methodology to the point where it is close to the Monte Carlo method. Although the Monte Carlo method is still more accurate, it should be mentioned that it could be computationally time consuming. The longer the tenor, the longer it will take to estimate the EE. Taking all the simplifying assumptions in this study into account, the time to estimate the EE takes relatively long when compared to the CEM. An important note is that the R code used to estimate the EE was not necessarily written in the most efficient manner and can be greatly improved. More efficient code will improve the computational time drastically.

CHAPTER 5
CONCLUSION AND OPEN QUESTIONS

There are various methods to estimate expected exposure (EE) to calculate Credit Valuation Adjustment (CVA). CVA is essentially an adjustment to the measurement of derivative assets to reflect the default risk of the counterparty. The author compared the Current Exposure Method (CEM), and expansions thereof, with the Monte Carlo Method to estimate the EE for an interest rate swap (IRS).

In Chapter 2 a literature review was done on the two methods to calculate the EE, as well as a discussion on the other elements needed to calculate CVA such as probability of default (PD) and loss given default (LGD). For the Monte Carlo method it was necessary to explore interest rate models and how to simulate the short rate. Further, the theory on IRS pricing and valuation was also discussed. Ways to mitigate credit exposure were also briefly touched upon. The methodology used to implement the theory was then described in Chapter 3.

The results for estimating EE with the CEM methods and the Monte Carlo method are represented in Chapter 4. The CEM methods are computationally much simpler than the Monte Carlo method and this is evident in the time the estimation of EE for each method took. CEM methods can be almost instant, whilst the Monte Carlo method can take anywhere from a few seconds to a few hours. What should be noted again is that the statistical programming code used in this study was written completely by the author and can be regarded as inefficient programming. Therefore, the computational time of the Monte Carlo method can be improved greatly. That being said, the efficiency of the code had no impact on the accuracy of the results. The code was written in the statistical programming package R and is represented in Appendix C.

Although the initial research objective was achieved, there are still some open questions to be asked for future research. The research can possibly be extended to a portfolio of IRS's and even a portfolio consisting of various asset classes. When estimating the EE of a portfolio it is then also possible to study the effects of netting and collateralisation. A major area for further research is the effect that different short rate models and their calibration to the local market have on EE estimation using the Monte Carlo methodology. There are numerous recognised models for the short rate, some of which are very complex. Calibrating the model could improve how accurate the EE is estimated.

To calculate the CVA, the three main elements needed are the EE, PD and the LGD. PD and LGD are in themselves very complex and their effect on CVA can be research topics on their own. One important assumption made in this study was that the effect of wrong-way risk was ignored. Wrong-way risk is the term used for the dependence between exposure and counterparty credit quality. When exposure is high, the counterparty is more likely to default. Therefore, the most interesting open question is how to quantify wrong-way risk and what the effect of wrong-way risk can have on CVA.

Finally, these results show that CEM Method 1 and Method 2 are relatively inaccurate compared to Method 3. Considering the resources smaller non-financial organisations might have, it is the author's opinion that Method 3 is a suitable replacement for the Monte Carlo method, although the Monte Carlo method is still notably more accurate.

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APPENDIX A: MATHEMATICAL RESULTS

A.1 MARTINGALE

Definition (Martingale) A sequence of random variables $X_0, X_1, \dots$ is a *martingale* if, for all $i > 0$,

$$E(X_i | X_{i-1}, X_{i-2}, \dots, X_0) = X_{i-1},$$

where E denotes expectation (Hull, 2009:620).

A.2 EQUIVALENT MARTINGALE MEASURE

A price process N_t is a numeraire if for all $t \in [0, T]$, the prices are strictly positive.

Definition (Equivalent Martingale Measure) Suppose N is a numeraire. A measure $\mathbb{Q}$ on (Ω, F) is a Equivalent Martingale Measure for numeraire N if and only if

1. $\mathbb{Q}$ is equivalent to $\mathbb{P}$ i.e. both measures have the same null sets,

2. $\hat{S}_t = \frac{S_t}{N_t}$ is a $\mathbb{Q}$-martingale

(Jones, 2010:18).

A.3 RISK-NEUTRAL VALUATION

Theorem (Risk-Neutral Valuation) Suppose that X is an attainable contingent claim, and that $\mathbb{Q}$ is an Equivalent Martingale Measure for numeraire N. Then,

$$\hat{X}_t = E^{\mathbb{Q}}\left[\hat{X}_T | \mathrm{F}_t\right],$$

i.e.,

$$X_t = E^{\mathbb{Q}}\left[\frac{X_T}{N_T} | \mathrm{F}_t\right]$$

(Jones, 2010:18).

APPENDIX B: TABLES AND FIGURES

Table B.1: Exposure Profiles for 2-Year Swaps

	Swap Rate								
	0.04			0.08			0.12		
Time	**Monte Carlo**	**CEM: Method1**	**CEM: Method 3**	**Monte Carlo**	**CEM: Method1**	**CEM: Method 3**	**Monte Carlo**	**CEM: Method1**	**CEM: Method 3**
0.00	0.00000	0.00500	0.00500	0.04691	0.05191	0.05191	0.12223	0.12723	0.12723
0.25	0.01047	0.00500	0.00438	0.04739	0.05191	0.04542	0.10673	0.12723	0.11132
0.50	0.01536	0.00500	0.00375	0.04590	0.05191	0.03893	0.09316	0.12723	0.09542
0.75	0.01687	0.00500	0.00313	0.04152	0.05191	0.03244	0.07911	0.12723	0.07952
1.00	0.01721	0.00500	0.00250	0.03597	0.05191	0.02595	0.06484	0.12723	0.06361
1.25	0.01475	0.00500	0.00188	0.02886	0.05191	0.01947	0.05007	0.12723	0.04771
1.50	0.01116	0.00500	0.00125	0.02076	0.05191	0.01298	0.03421	0.12723	0.03181
1.75	0.00627	0.00500	0.00063	0.01122	0.05191	0.00649	0.01760	0.12723	0.01590
2.00	0.00000	0.00500	0.00000	0.00000	0.05191	0.00000	0.00000	0.12723	0.00000

Table B.2: Exposure Profiles for 10-Year Swaps

	Swap Rate								
	0.04			0.08			0.12		
Time	Monte Carlo	CEM: Method1	CEM: Method 3	Monte Carlo	CEM: Method1	CEM: Method 3	Monte Carlo	CEM: Method1	CEM: Method 3
0.00	0.00000	0.01500	0.01500	0.19338	0.20838	0.20838	0.50156	0.51656	0.51656
0.25	0.00231	0.01500	0.01463	0.19017	0.20838	0.20317	0.49002	0.51656	0.50365
0.50	0.00679	0.01500	0.01425	0.18498	0.20838	0.19796	0.47814	0.51656	0.49073
0.75	0.00987	0.01500	0.01388	0.18129	0.20838	0.19275	0.46605	0.51656	0.47782
1.00	0.01310	0.01500	0.01350	0.17816	0.20838	0.18754	0.45409	0.51656	0.46491
1.25	0.01492	0.01500	0.01313	0.17476	0.20838	0.18233	0.44142	0.51656	0.45199
1.50	0.01582	0.01500	0.01275	0.17129	0.20838	0.17712	0.42913	0.51656	0.43908
1.75	0.01733	0.01500	0.01238	0.16931	0.20838	0.17191	0.41754	0.51656	0.42616
2.00	0.01870	0.01500	0.01200	0.16527	0.20838	0.16670	0.40540	0.51656	0.41325
2.25	0.01953	0.01500	0.01163	0.16251	0.20838	0.16149	0.39186	0.51656	0.40034
2.50	0.02049	0.01500	0.01125	0.15799	0.20838	0.15628	0.38111	0.51656	0.38742
2.75	0.02101	0.01500	0.01088	0.15410	0.20838	0.15107	0.37085	0.51656	0.37451
3.00	0.02172	0.01500	0.01050	0.14977	0.20838	0.14587	0.35821	0.51656	0.36159
3.25	0.02240	0.01500	0.01013	0.14555	0.20838	0.14066	0.34611	0.51656	0.34868
3.50	0.02302	0.01500	0.00975	0.14141	0.20838	0.13545	0.33312	0.51656	0.33576
3.75	0.02389	0.01500	0.00938	0.13630	0.20838	0.13024	0.32301	0.51656	0.32285
4.00	0.02436	0.01500	0.00900	0.13274	0.20838	0.12503	0.31103	0.51656	0.30994
4.25	0.02498	0.01500	0.00863	0.12791	0.20838	0.11982	0.30087	0.51656	0.29702
4.50	0.02508	0.01500	0.00825	0.12249	0.20838	0.11461	0.28707	0.51656	0.28411
4.75	0.02542	0.01500	0.00788	0.11913	0.20838	0.10940	0.27525	0.51656	0.27119
5.00	0.02509	0.01500	0.00750	0.11494	0.20838	0.10419	0.26127	0.51656	0.25828
5.25	0.02584	0.01500	0.00713	0.11171	0.20838	0.09898	0.24987	0.51656	0.24537
5.50	0.02575	0.01500	0.00675	0.10825	0.20838	0.09377	0.23864	0.51656	0.23245
5.75	0.02666	0.01500	0.00638	0.10409	0.20838	0.08856	0.22650	0.51656	0.21954
6.00	0.02663	0.01500	0.00600	0.09990	0.20838	0.08335	0.21465	0.51656	0.20662
6.25	0.02683	0.01500	0.00563	0.09553	0.20838	0.07814	0.20367	0.51656	0.19371
6.50	0.02719	0.01500	0.00525	0.09181	0.20838	0.07293	0.19158	0.51656	0.18080
6.75	0.02716	0.01500	0.00488	0.08737	0.20838	0.06772	0.17799	0.51656	0.16788
7.00	0.02720	0.01500	0.00450	0.08296	0.20838	0.06251	0.16635	0.51656	0.15497
7.25	0.02731	0.01500	0.00413	0.07742	0.20838	0.05730	0.15385	0.51656	0.14205
7.50	0.02670	0.01500	0.00375	0.07243	0.20838	0.05209	0.14053	0.51656	0.12914
7.75	0.02616	0.01500	0.00338	0.06754	0.20838	0.04689	0.12770	0.51656	0.11623
8.00	0.02559	0.01500	0.00300	0.06264	0.20838	0.04168	0.11555	0.51656	0.10331
8.25	0.02423	0.01500	0.00263	0.05701	0.20838	0.03647	0.10270	0.51656	0.09040
8.50	0.02309	0.01500	0.00225	0.05086	0.20838	0.03126	0.08947	0.51656	0.07748
8.75	0.02092	0.01500	0.00188	0.04490	0.20838	0.02605	0.07570	0.51656	0.06457
9.00	0.01869	0.01500	0.00150	0.03690	0.20838	0.02084	0.06158	0.51656	0.05166
9.25	0.01545	0.01500	0.00113	0.02903	0.20838	0.01563	0.04741	0.51656	0.03874
9.50	0.01127	0.01500	0.00075	0.02050	0.20838	0.01042	0.03266	0.51656	0.02583
9.75	0.00623	0.01500	0.00038	0.01081	0.20838	0.00521	0.01706	0.51656	0.01291
10.00	0.00000	0.01500	0.00000	0.00000	0.20838	0.00000	0.00000	0.51656	0.00000

Table B.3: Exposure Profiles for 25-Year Swaps

	Swap Rate								
	0.04			0.08			0.12		
Time	**Monte Carlo**	**CEM: Method1**	**CEM: Method 3**	**Monte Carlo**	**CEM: Method1**	**CEM: Method 3**	**Monte Carlo**	**CEM: Method1**	**CEM: Method 3**
0.00	0.00000	0.01500	0.01500	0.35329	0.36829	0.36829	0.89882	0.91382	0.91382
0.25	0.00008	0.01500	0.01485	0.35105	0.36829	0.36461	0.89192	0.91382	0.90468
0.50	0.00086	0.01500	0.01470	0.34772	0.36829	0.36092	0.88486	0.91382	0.89554
0.75	0.00184	0.01500	0.01455	0.34322	0.36829	0.35724	0.87851	0.91382	0.88640
1.00	0.00269	0.01500	0.01440	0.33899	0.36829	0.35356	0.86989	0.91382	0.87727
1.25	0.00355	0.01500	0.01425	0.33687	0.36829	0.34987	0.85879	0.91382	0.86813
1.50	0.00406	0.01500	0.01410	0.33304	0.36829	0.34619	0.85184	0.91382	0.85899
1.75	0.00458	0.01500	0.01395	0.32888	0.36829	0.34251	0.84532	0.91382	0.84985
2.00	0.00472	0.01500	0.01380	0.32509	0.36829	0.33883	0.83760	0.91382	0.84071
2.25	0.00517	0.01500	0.01365	0.32205	0.36829	0.33514	0.82861	0.91382	0.83158
2.50	0.00504	0.01500	0.01350	0.31790	0.36829	0.33146	0.82507	0.91382	0.82244
2.75	0.00491	0.01500	0.01335	0.31488	0.36829	0.32778	0.81800	0.91382	0.81330
3.00	0.00505	0.01500	0.01320	0.31225	0.36829	0.32409	0.81169	0.91382	0.80416
3.25	0.00538	0.01500	0.01305	0.31121	0.36829	0.32041	0.80454	0.91382	0.79502
3.50	0.00615	0.01500	0.01290	0.30927	0.36829	0.31673	0.79915	0.91382	0.78588
3.75	0.00618	0.01500	0.01275	0.30755	0.36829	0.31305	0.79244	0.91382	0.77675
4.00	0.00616	0.01500	0.01260	0.30460	0.36829	0.30936	0.78738	0.91382	0.76761
4.25	0.00638	0.01500	0.01245	0.30245	0.36829	0.30568	0.78228	0.91382	0.75847
4.50	0.00639	0.01500	0.01230	0.29961	0.36829	0.30200	0.77598	0.91382	0.74933
4.75	0.00637	0.01500	0.01215	0.29715	0.36829	0.29831	0.77164	0.91382	0.74019
5.00	0.00631	0.01500	0.01200	0.29514	0.36829	0.29463	0.76769	0.91382	0.73106
5.25	0.00654	0.01500	0.01185	0.29272	0.36829	0.29095	0.76316	0.91382	0.72192
5.50	0.00651	0.01500	0.01170	0.28989	0.36829	0.28727	0.75977	0.91382	0.71278
5.75	0.00682	0.01500	0.01155	0.28884	0.36829	0.28358	0.75530	0.91382	0.70364
6.00	0.00671	0.01500	0.01140	0.28483	0.36829	0.27990	0.74782	0.91382	0.69450
6.25	0.00674	0.01500	0.01125	0.28236	0.36829	0.27622	0.74122	0.91382	0.68536
6.50	0.00665	0.01500	0.01110	0.28213	0.36829	0.27253	0.73412	0.91382	0.67623
6.75	0.00690	0.01500	0.01095	0.28228	0.36829	0.26885	0.72760	0.91382	0.66709
7.00	0.00696	0.01500	0.01080	0.27934	0.36829	0.26517	0.72104	0.91382	0.65795
7.25	0.00666	0.01500	0.01065	0.27547	0.36829	0.26148	0.71542	0.91382	0.64881
7.50	0.00682	0.01500	0.01050	0.27209	0.36829	0.25780	0.71123	0.91382	0.63967
7.75	0.00744	0.01500	0.01035	0.26739	0.36829	0.25412	0.70582	0.91382	0.63053
8.00	0.00756	0.01500	0.01020	0.26418	0.36829	0.25044	0.69951	0.91382	0.62140
8.25	0.00792	0.01500	0.01005	0.26148	0.36829	0.24675	0.69162	0.91382	0.61226
8.50	0.00817	0.01500	0.00990	0.25911	0.36829	0.24307	0.68424	0.91382	0.60312
8.75	0.00821	0.01500	0.00975	0.25735	0.36829	0.23939	0.67601	0.91382	0.59398
9.00	0.00836	0.01500	0.00960	0.25644	0.36829	0.23570	0.66920	0.91382	0.58484
9.25	0.00841	0.01500	0.00945	0.25548	0.36829	0.23202	0.66136	0.91382	0.57571
9.50	0.00877	0.01500	0.00930	0.25386	0.36829	0.22834	0.65559	0.91382	0.56657
9.75	0.00883	0.01500	0.00915	0.25041	0.36829	0.22466	0.64897	0.91382	0.55743
10.00	0.00906	0.01500	0.00900	0.24696	0.36829	0.22097	0.64124	0.91382	0.54829

Swap Rate									
	0.04			0.08			0.12		
Time	Monte Carlo	CEM: Method1	CEM: Method 3	Monte Carlo	CEM: Method1	CEM: Method 3	Monte Carlo	CEM: Method1	CEM: Method 3
10.25	0.00944	0.01500	0.00885	0.24438	0.36829	0.21729	0.63477	0.91382	0.53915
10.50	0.00979	0.01500	0.00870	0.24316	0.36829	0.21361	0.62603	0.91382	0.53001
10.75	0.00990	0.01500	0.00855	0.23866	0.36829	0.20992	0.61811	0.91382	0.52088
11.00	0.01056	0.01500	0.00840	0.23753	0.36829	0.20624	0.61068	0.91382	0.51174
11.25	0.01036	0.01500	0.00825	0.23528	0.36829	0.20256	0.60384	0.91382	0.50260
11.50	0.01071	0.01500	0.00810	0.23227	0.36829	0.19888	0.59595	0.91382	0.49346
11.75	0.01114	0.01500	0.00795	0.22895	0.36829	0.19519	0.59079	0.91382	0.48432
12.00	0.01162	0.01500	0.00780	0.22499	0.36829	0.19151	0.58143	0.91382	0.47519
12.25	0.01225	0.01500	0.00765	0.22158	0.36829	0.18783	0.57255	0.91382	0.46605
12.50	0.01239	0.01500	0.00750	0.22009	0.36829	0.18414	0.56408	0.91382	0.45691
12.75	0.01297	0.01500	0.00735	0.21714	0.36829	0.18046	0.55567	0.91382	0.44777
13.00	0.01327	0.01500	0.00720	0.21431	0.36829	0.17678	0.54597	0.91382	0.43863
13.25	0.01387	0.01500	0.00705	0.21057	0.36829	0.17310	0.53479	0.91382	0.42949
13.50	0.01403	0.01500	0.00690	0.20739	0.36829	0.16941	0.52434	0.91382	0.42036
13.75	0.01393	0.01500	0.00675	0.20374	0.36829	0.16573	0.51555	0.91382	0.41122
14.00	0.01446	0.01500	0.00660	0.19984	0.36829	0.16205	0.50746	0.91382	0.40208
14.25	0.01483	0.01500	0.00645	0.19636	0.36829	0.15836	0.49911	0.91382	0.39294
14.50	0.01508	0.01500	0.00630	0.19265	0.36829	0.15468	0.49027	0.91382	0.38380
14.75	0.01516	0.01500	0.00615	0.18737	0.36829	0.15100	0.48206	0.91382	0.37467
15.00	0.01518	0.01500	0.00600	0.18584	0.36829	0.14732	0.47316	0.91382	0.36553
15.25	0.01569	0.01500	0.00585	0.18311	0.36829	0.14363	0.46371	0.91382	0.35639
15.50	0.01682	0.01500	0.00570	0.17926	0.36829	0.13995	0.45346	0.91382	0.34725
15.75	0.01696	0.01500	0.00555	0.17606	0.36829	0.13627	0.44494	0.91382	0.33811
16.00	0.01716	0.01500	0.00540	0.17102	0.36829	0.13258	0.43482	0.91382	0.32897
16.25	0.01741	0.01500	0.00525	0.16768	0.36829	0.12890	0.42598	0.91382	0.31984
16.50	0.01788	0.01500	0.00510	0.16496	0.36829	0.12522	0.41763	0.91382	0.31070
16.75	0.01811	0.01500	0.00495	0.16029	0.36829	0.12154	0.40808	0.91382	0.30156
17.00	0.01873	0.01500	0.00480	0.15675	0.36829	0.11785	0.39657	0.91382	0.29242
17.25	0.01916	0.01500	0.00465	0.15357	0.36829	0.11417	0.38514	0.91382	0.28328
17.50	0.01965	0.01500	0.00450	0.14947	0.36829	0.11049	0.37510	0.91382	0.27415
17.75	0.02043	0.01500	0.00435	0.14726	0.36829	0.10680	0.36456	0.91382	0.26501
18.00	0.02157	0.01500	0.00420	0.14253	0.36829	0.10312	0.35394	0.91382	0.25587
18.25	0.02162	0.01500	0.00405	0.13849	0.36829	0.09944	0.34409	0.91382	0.24673
18.50	0.02269	0.01500	0.00390	0.13584	0.36829	0.09576	0.33212	0.91382	0.23759
18.75	0.02365	0.01500	0.00375	0.13305	0.36829	0.09207	0.31986	0.91382	0.22845
19.00	0.02375	0.01500	0.00360	0.12918	0.36829	0.08839	0.30770	0.91382	0.21932
19.25	0.02459	0.01500	0.00345	0.12663	0.36829	0.08471	0.29736	0.91382	0.21018
19.50	0.02459	0.01500	0.00330	0.12322	0.36829	0.08102	0.28651	0.91382	0.20104
19.75	0.02500	0.01500	0.00315	0.11808	0.36829	0.07734	0.27504	0.91382	0.19190
20.00	0.02519	0.01500	0.00300	0.11336	0.36829	0.07366	0.26342	0.91382	0.18276
20.25	0.02564	0.01500	0.00285	0.10940	0.36829	0.06997	0.25098	0.91382	0.17363

Swap Rate									
	0.04			0.08			0.12		
Time	Monte Carlo	CEM: Method1	CEM: Method 3	Monte Carlo	CEM: Method1	CEM: Method 3	Monte Carlo	CEM: Method1	CEM: Method 3
20.50	0.02645	0.01500	0.00270	0.10533	0.36829	0.06629	0.23903	0.91382	0.16449
20.75	0.02657	0.01500	0.00255	0.10246	0.36829	0.06261	0.22721	0.91382	0.15535
21.00	0.02712	0.01500	0.00240	0.09775	0.36829	0.05893	0.21622	0.91382	0.14621
21.25	0.02750	0.01500	0.00225	0.09374	0.36829	0.05524	0.20433	0.91382	0.13707
21.50	0.02742	0.01500	0.00210	0.09019	0.36829	0.05156	0.19261	0.91382	0.12793
21.75	0.02771	0.01500	0.00195	0.08665	0.36829	0.04788	0.18038	0.91382	0.11880
22.00	0.02795	0.01500	0.00180	0.08220	0.36829	0.04419	0.16829	0.91382	0.10966
22.25	0.02683	0.01500	0.00165	0.07820	0.36829	0.04051	0.15652	0.91382	0.10052
22.50	0.02690	0.01500	0.00150	0.07295	0.36829	0.03683	0.14231	0.91382	0.09138
22.75	0.02670	0.01500	0.00135	0.06744	0.36829	0.03315	0.12947	0.91382	0.08224
23.00	0.02579	0.01500	0.00120	0.06125	0.36829	0.02946	0.11681	0.91382	0.07311
23.25	0.02488	0.01500	0.00105	0.05545	0.36829	0.02578	0.10431	0.91382	0.06397
23.50	0.02322	0.01500	0.00090	0.04948	0.36829	0.02210	0.09115	0.91382	0.05483
23.75	0.02077	0.01500	0.00075	0.04324	0.36829	0.01841	0.07645	0.91382	0.04569
24.00	0.01823	0.01500	0.00060	0.03611	0.36829	0.01473	0.06229	0.91382	0.03655
24.25	0.01503	0.01500	0.00045	0.02838	0.36829	0.01105	0.04778	0.91382	0.02741
24.50	0.01111	0.01500	0.00030	0.02002	0.36829	0.00737	0.03266	0.91382	0.01828
24.75	0.00621	0.01500	0.00015	0.01050	0.36829	0.00368	0.01680	0.91382	0.00914
25.00	0.00000	0.01500	0.00000	0.00000	0.36829	0.00000	0.00000	0.91382	0.00000

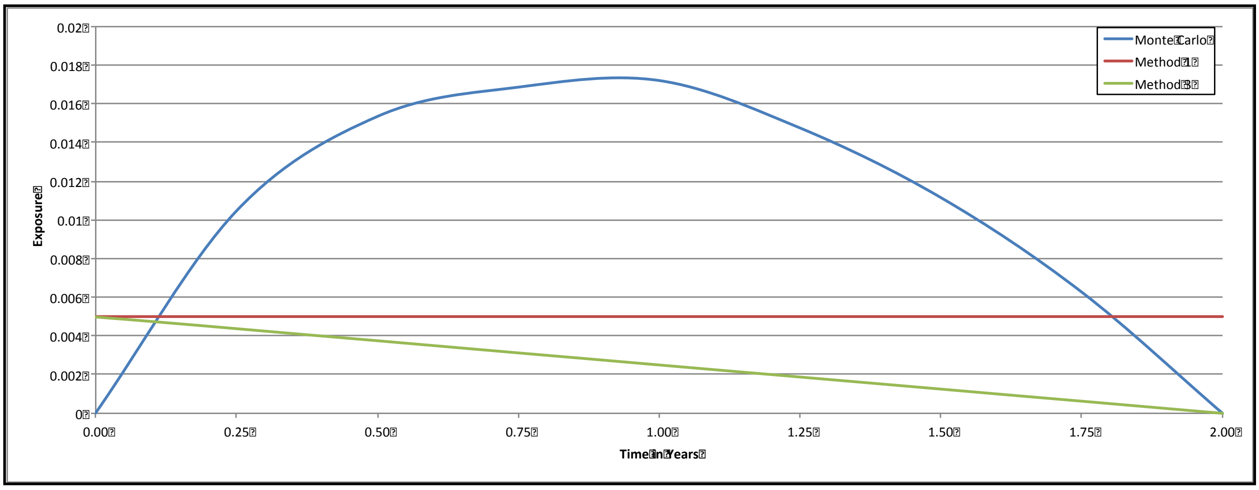

Figure B.1: Exposure Profiles for the 4% 2-Year IRS for different methods

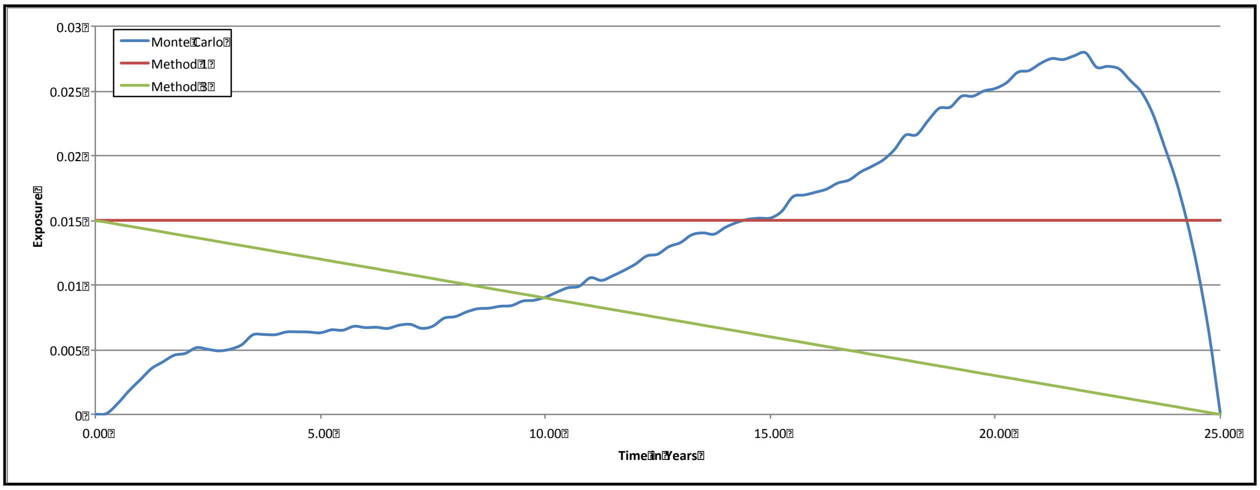

Figure B.2: Exposure Profiles for the 4% 25-Year IRS for different methods

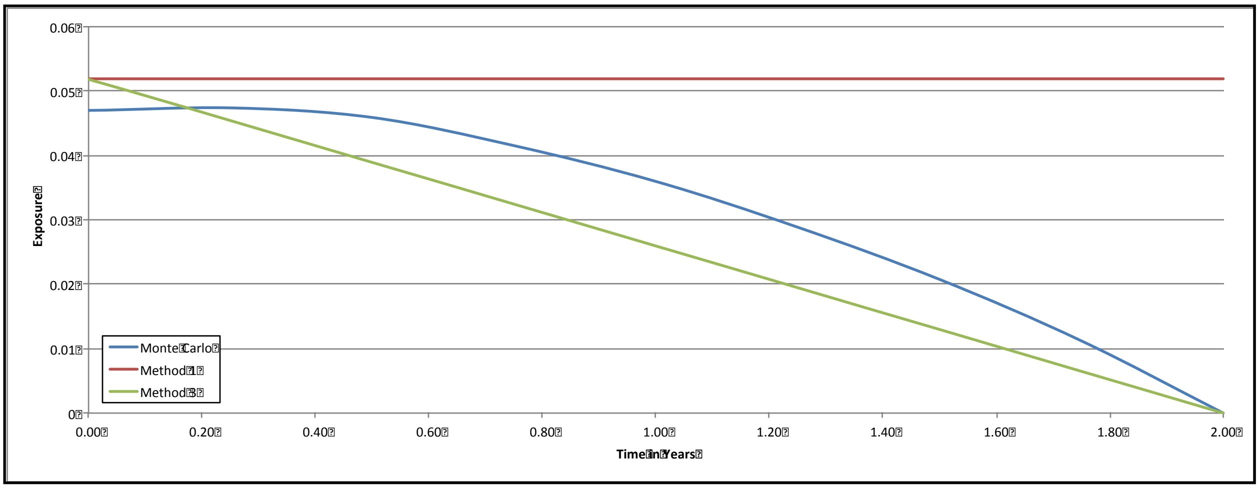

Figure B.3: Exposure Profiles for the 8% 2-Year IRS for different methods

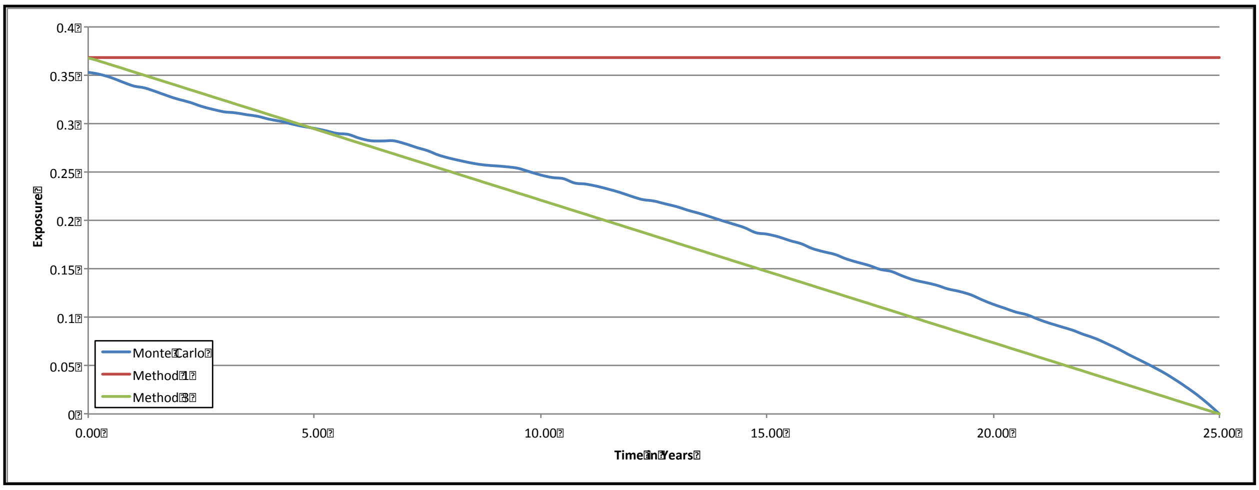

Figure B.4: Exposure Profiles for the 8% 25-Year IRS for different methods

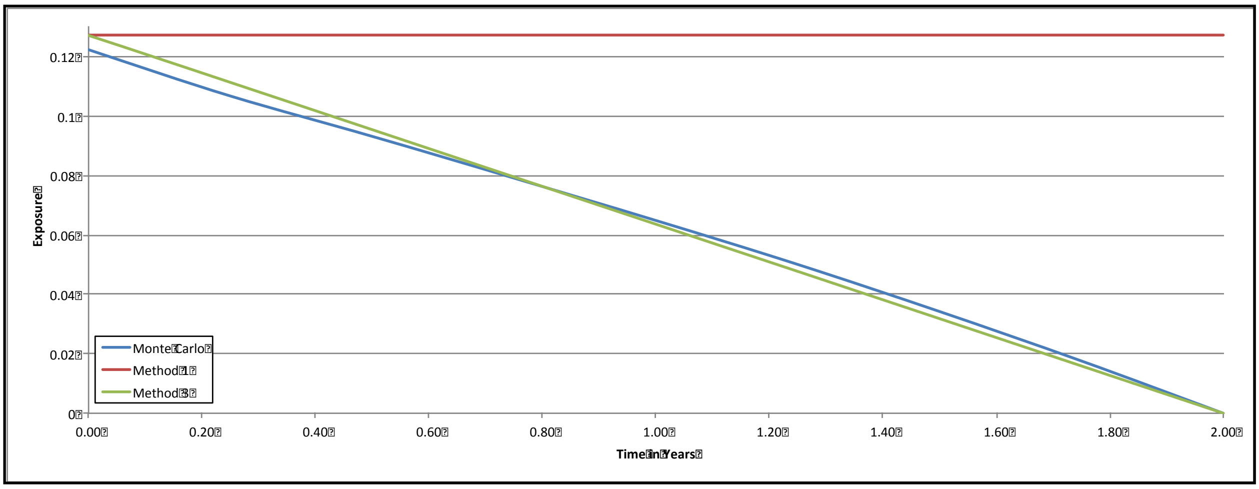

Figure B.5: Exposure Profiles for the 12% 2-Year IRS for different methods

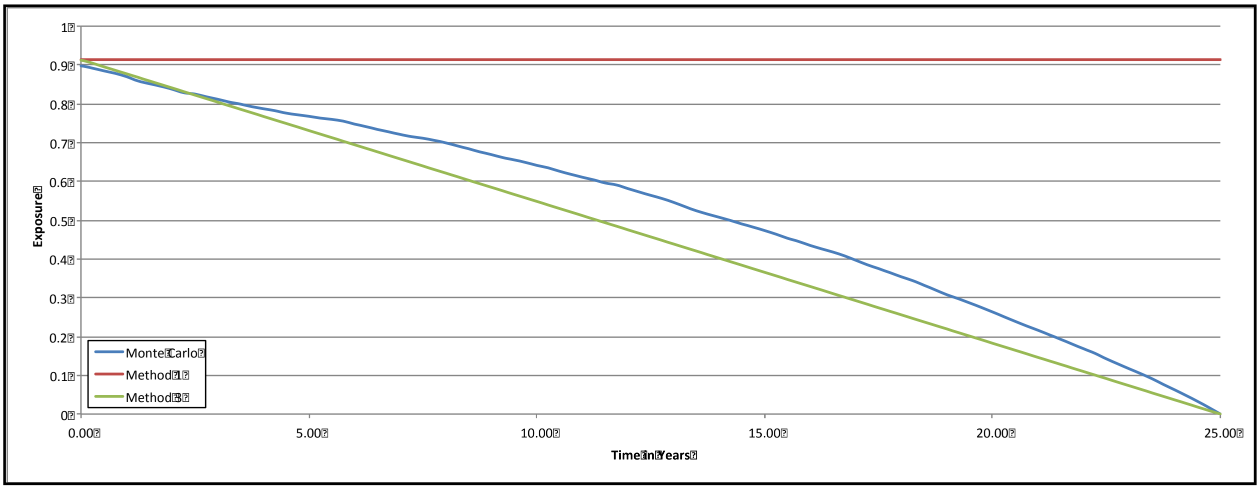

Figure B.6: Exposure Profiles for the 12% 25-Year IRS for different methods

APPENDIX C: R CODE

```
#VASICEK MODEL
VasicekZeroRates<-function(a,b,sigma,r0,T,dt,n)
{
    t<-c(seq(0,T,(1/252)))
    T.min.t<-T-rev(t)
    length<-(T*252)+1

    shortrate<-function(r)
    {
        temp1<-a*(b-r)*(1/252)
        temp2<-sigma*rnorm(1)*sqrt(1/252)
        rt<-r+temp1+temp2
        return(rt)
    }

    f<-matrix(r0,nrow=(T*252)+1,ncol=n)

    for(j in 1:n)
    {
        for(i in 1:(T*252)+1)
        {
            f[i,j]<-shortrate(f[i-1,j])
        }
    }
    #return(f)

    Bvec<-(1-exp(-a*(T.min.t)))/a
    #return(Bvec)

    Avec<-exp((((Bvec-T.min.t)*(((a^2)*b)-(sigma^2)/2))/(a^2))-
    (((sigma^2)*(Bvec^2))/(4*a)))
    #return(Avec)

    Bmat<-matrix(NA,ncol=length,nrow=length)

    for(i in 1:length)
        {
        Bmat[,i]<-Bvec*f[i]
        }
    #return(Bmat)

        #Calculate the zero rates at time t=0,0.25,...for ith maturity
    i=dt,2dt,...,T
    temp1<-log(Avec)
    temp2<-Bmat*(1/T.min.t)
    temp3<-(1/T.min.t)*temp1
    Rmat<-temp2-temp3
    Rmat[1,]<-0
    Rmat<-Rmat[,-length(Bvec)]
    Rmat<-Rmat[seq(1,nrow(Rmat),63),seq(1,ncol(Rmat),63)]
    return(Rmat)

    #Rmat[1,1]<-NA
```

```
    #plot(x=seq(0,T,dt),y=Rmat[,1],type="l",
    #col="mediumorchid2",xlab="Time in Years",ylab="R(0,T)",
    #       main="Vasicek Zero Curve")

}

#FORWARD RATES
ForwardRates<-function(zcurve,T,dt)
{
    t<-c(seq(0,T,dt))
    T.min.t<-T-rev(t)

    FR<-NULL
    for(i in 1:length(zcurve)-1)
    {
        FR[i]<-((zcurve[i+1]*t[i+1])-(zcurve[i]*t[i]))/dt
    }

    temp1<-FR*dt

    FRmat<-exp(temp1)-1 #compounded dt'ly

 }

#DISCOUNTING COUPONS
Discount<-function(coupmat,zcurve,T,dt)
{
    t<-c(seq(0,T,dt))
    T.min.t<-T-rev(t)

    DFmat<-exp(-zcurve*t)
    #return(DFmat)

    if (is.vector(DFmat)==TRUE) PVmat<-DFmat[2:length(DFmat)]*coupmat

    else PVmat<-DFmat[2:nrow(DFmat),]*coupmat

    return(PVmat)

 }

#PV COUPONS
PV<-function(PVmat)
{
    if (is.vector(PVmat)==TRUE) PV<-sum(PVmat)

    else PV<-apply(PVmat,2,sum)
    return(PV)
 }

#SWAP MTM
SwapMtM<-function(PVfixed,PVfloat,payfixed=1)
{
    if (payfixed==1) MtM<-PVfixed-PVfloat
    else MtM<-PVfloat-PVfixed
```

```
    MtM[length(MtM)+1]<-0
    return(MtM)
}

#SWAP VALUE
SwapValue<-function(a,b,sigma,swapvec,r0)
{
    swaprate<-swapvec[1]
    T<-swapvec[2]
    payfixed<-swapvec[3]
    spread<-swapvec[4]
    dt<-swapvec[5]

    t<-c(seq(0,T,dt))

    ZC<-VasicekZeroRates(a=a,b=b,sigma=sigma,r0=r0,T=T,dt=dt,n=1)
    ZC1<-ZC+spread

    Float<-apply(ZC1,2,ForwardRates,T=T,dt=dt)
    Float<-Float[,ncol(Float):1]
    Float[lower.tri(Float)]<-0
    Float<-Float[,ncol(Float):1]

    Fixed<-
    matrix(c(rep(swaprate*dt,times=(T/dt)^2)),nrow=T/dt,ncol=T/dt)
    Fixed<-Fixed[,ncol(Fixed):1]
    Fixed[lower.tri(Fixed)]<-0
    Fixed<-Fixed[,ncol(Fixed):1]

    Discfloat<-Discount(coupmat=Float,zcurve=ZC,T=T,dt=dt)
    Discfixed<-Discount(coupmat=Fixed,zcurve=ZC,T=T,dt=dt)

    PVfloat<-apply(Discfloat,2,sum)
    PVfixed<-apply(Discfixed,2,sum)

    MtMValue<-
    SwapMtM(PVfixed=PVfixed,PVfloat=PVfloat,payfixed=payfixed)
    return(MtMValue)

    #plot(x=t,y=MtMValue,type="l")
 }

#Plot van MtM Values, EE en PFE
Exposure<-function(a,b,sigma,swapvec,r0,n)
{
    Time.elapsed<-proc.time()[3]
    T<-swapvec[2]
    dt<-swapvec[5]

    FutureValues<-matrix(NA,nrow=(T/dt)+1,ncol=n)
    for(i in 1:n)
    {
        FutureValues[,i]<-
        SwapValue(a=a,b=b,sigma=sigma,swapvec=swapvec,r0=r0)
    }
```

```
    FutureValuesTemp<-FutureValues
    FutureValuesTemp2<-apply(FutureValues,1,sort)
    FutureValuesTemp[FutureValuesTemp<0]<-0

    EE<-(apply(FutureValuesTemp,1,sum))/n
    #return(EE)

    PFE<-(FutureValuesTemp2[0.95*n,])
    E<-cbind(EE,PFE)

    Time.elapsed<-proc.time()[3]-Time.elapsed

    par(mfrow=c(2,1))
    plot<-matplot(x=c(seq(0,T,dt)),y=FutureValues,xlab="Time (Years)",
                  ylab="Future Simulated MtM Values of
    swap",type="l",lty=1,col=1:n,xaxs="i")
    abline(h=0,lwd=2)

    matplot(x=c(seq(0,T,dt)),y=E,xlab="Time
    (Years)",ylab="Exposure",type="l",lty=1,xaxs="i",col=1:(T/dt))
    abline(h=0,lwd=2)
    list(EE=EE,PFE=PFE,Time.elapsed=Time.elapsed)

    #return (plot)
}
```